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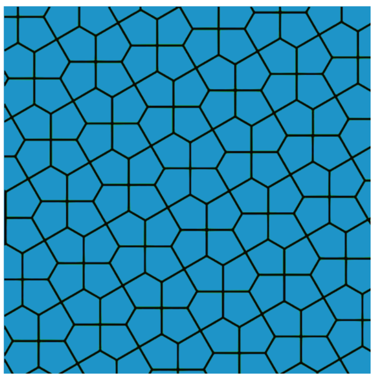
Unit 1, Lesson 1: Tiling the Plane

Let’s look at tiling patterns and think about area.

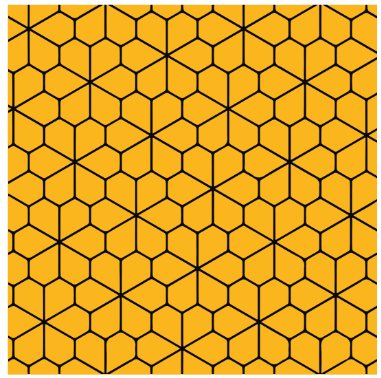
1.1: Which One Doesn’t Belong: Tilings

Which pattern doesn’t belong?

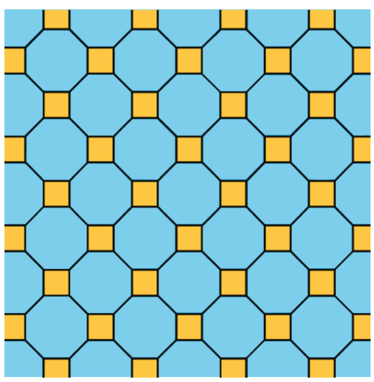
A



B



C



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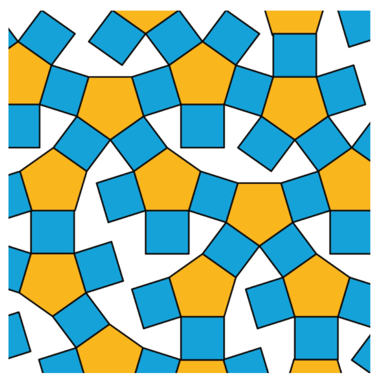

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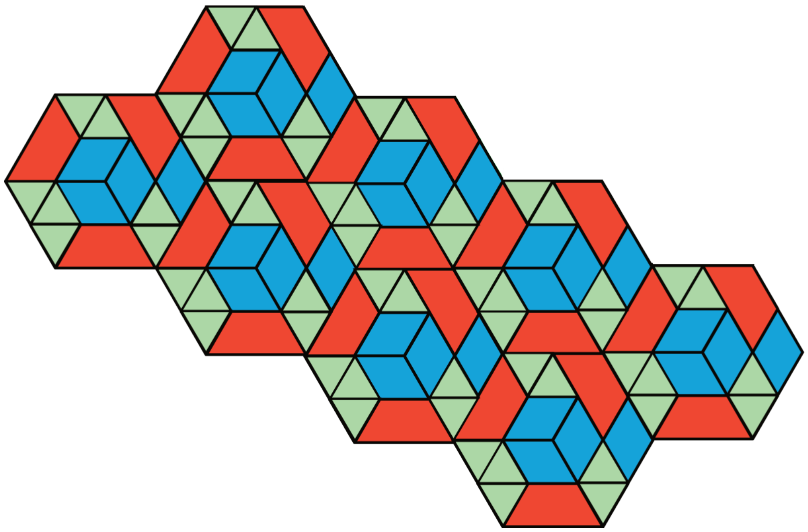
1.2: More Red, Green, or Blue?

m.openup.org/1/6-1-1-2

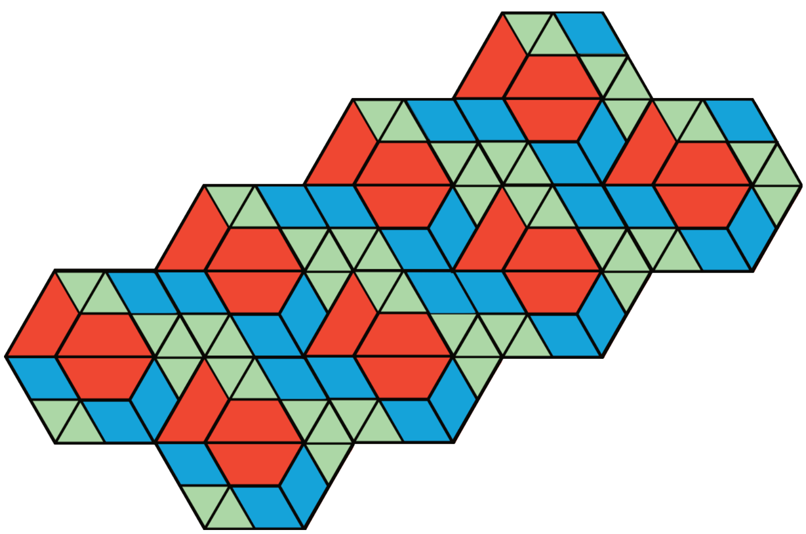
Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

Pattern A



Pattern B



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Are you ready for more?

On graph paper, create a tiling pattern so that:

- The pattern has at least two different shapes.
- The same amount of the plane is covered by each type of shape.

Lesson 1 Summary

In this lesson, we learned about *tiling* the plane, which means covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps.

Then, we compared tiling patterns and the shapes in them. In thinking about which patterns and shapes cover more of the plane, we have started to reason about **area**.

We will continue this work, and to learn how to use mathematical tools strategically to help us do mathematics.

Lesson 1 Glossary Terms

- area
- region

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Unit 1, Lesson 2: Finding Area by Decomposing and Rearranging

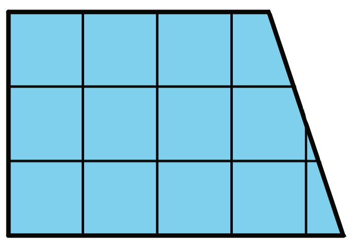
Let's create shapes and find their areas.

2.1: What is Area?

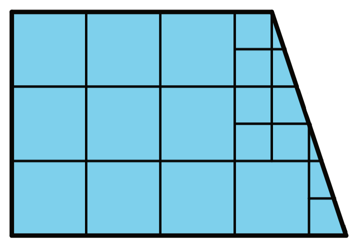
You may recall that the term **area** tells us something about the number of squares inside a two-dimensional shape.

1. Here are four drawings that each show squares inside a shape. Select **all** drawings whose squares could be used to find the area of the shape. Be prepared to explain your reasoning.

A



B



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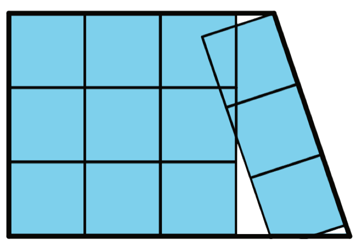
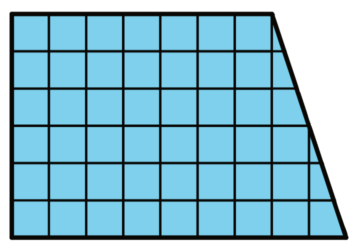


D



2. Write a definition of area that includes all the information that you think is important.

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2.2: Composing Shapes

m.openup.org/1/6-1-2-2

Your teacher will give you one square and some small, medium, and large right triangles. The area of the square is 1 square unit.

1. Notice that you can put together two small triangles to make a square. What is the area of the square composed of two small triangles? Be prepared to explain your reasoning.

2. Use your shapes to create a new shape with an area of 1 square unit that is not a square. Trace your shape.

3. Use your shapes to create a new shape with an area of 2 square units. Trace your shape.

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4. Use your shapes to create a *different* shape with an area of 2 square units. Trace your shape.

5. Use your shapes to create a new shape with an area of 4 square units. Trace your shape.

Are you ready for more?

Find a way to use all of your pieces to compose a single large square. What is the area of this large square?

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2.3: Tangram Triangles

m.openup.org/1/6-1-2-3

Recall that the area of the square you saw earlier is 1 square unit. Complete each statement and explain your reasoning.

1. The area of the small triangle is _________ square units. I know this because . . .

2. The area of the medium triangle is _________ square units. I know this because . . .

3. The area of the large triangle is _________ square units. I know this because . . .

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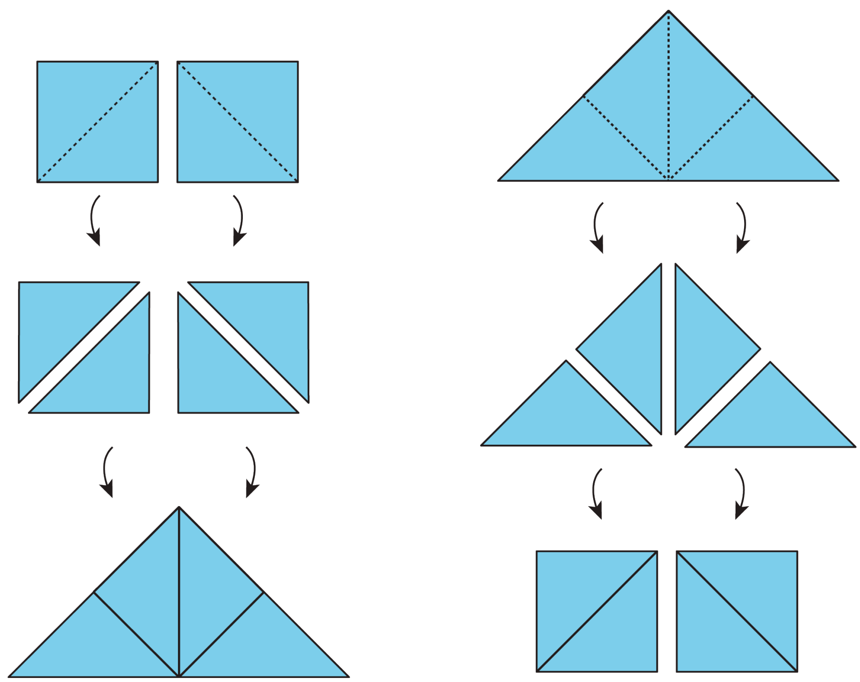
Lesson 2 Summary

Here are two important principles for finding **area**:

1. If two figures can be placed one on top of the other so that they match up exactly, then they have the *same area*.
2. We can **decompose** a figure (break a figure into pieces) and **rearrange** the pieces (move the pieces around) to find its area.

Here are illustrations of the two principles.



- Each square on the left can be decomposed into 2 triangles. These triangles can be rearranged into a large triangle. So the large triangle has the *same area* as the 2 squares.

- Similarly, the large triangle on the right can be decomposed into 4 equal triangles. The triangles can be rearranged to form 2 squares. If each square has an area of 1 square unit, then the area of the large triangle is 2 square units. We also can say that each small triangle has an area of $\frac{1}{2}$ square unit.

Lesson 2 Glossary Terms

- area

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- rearrange
- compose/decompose

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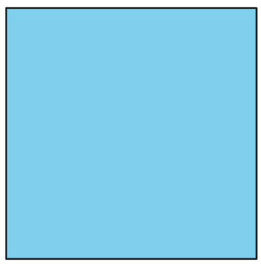
Unit 1, Lesson 3: Reasoning to Find Area

Let’s decompose and rearrange shapes to find their areas.

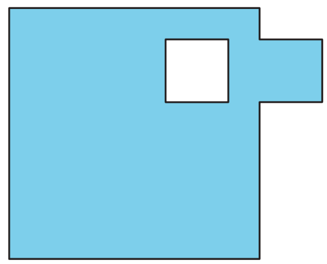
3.1: Comparing Regions

Is the area of Figure A greater than, less than, or equal to the area of the shaded region in Figure B? Be prepared to explain your reasoning.

A



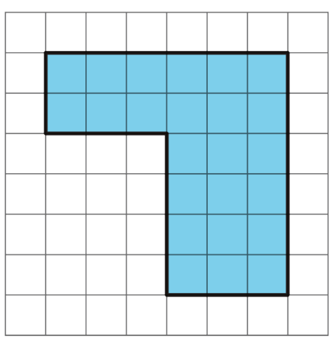
B



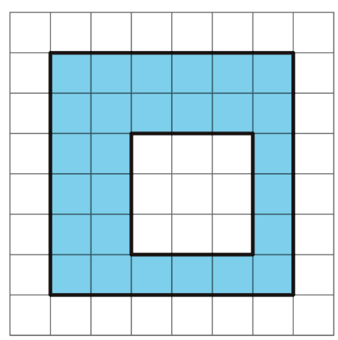
3.2: On the Grid

Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every square. Be prepared to explain your reasoning.

A



B



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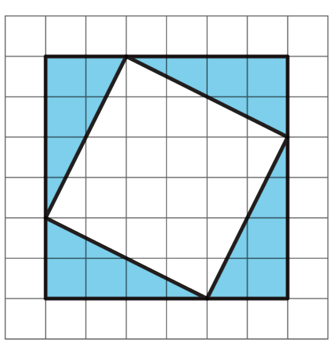

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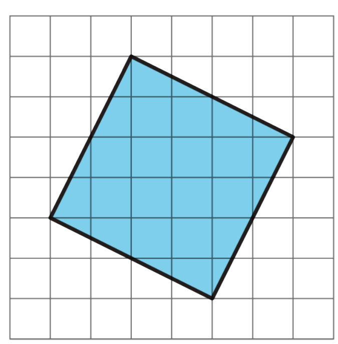

Are you ready for more?

Rearrange the triangles from Figure C so they fit inside Figure D. Draw and color a

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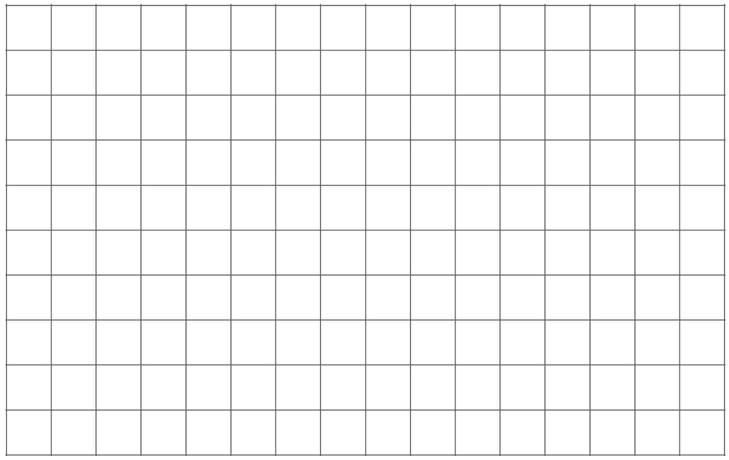
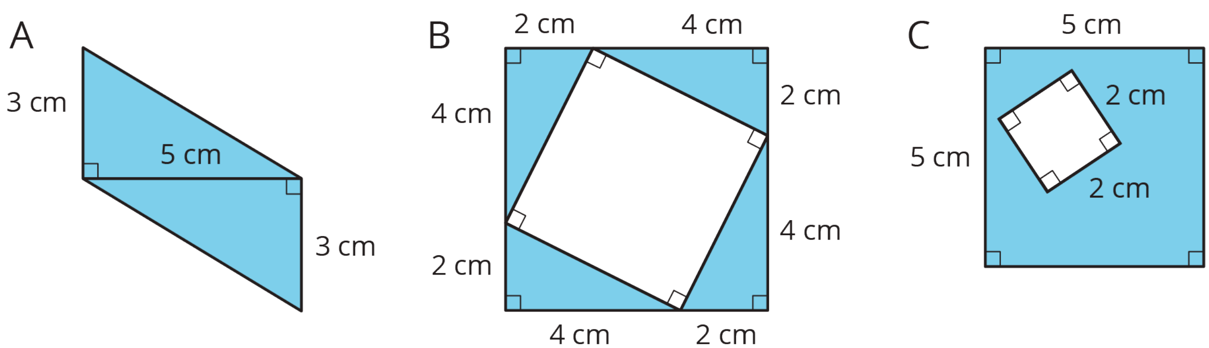
diagram of your work on the grid.



3.3: Off the Grid

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.



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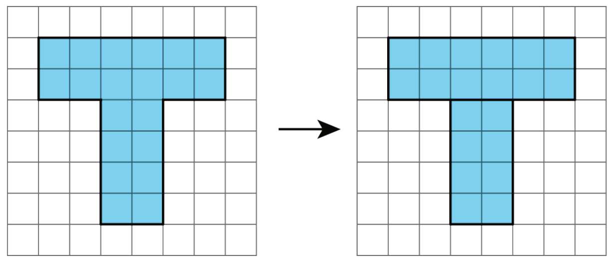
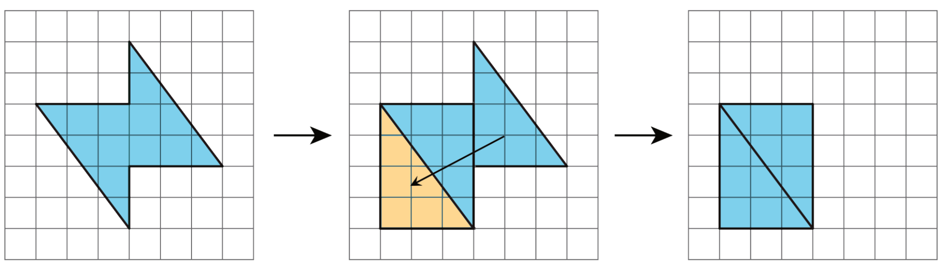
Lesson 3 Summary

There are different strategies we can use to find the area of a region. We can:

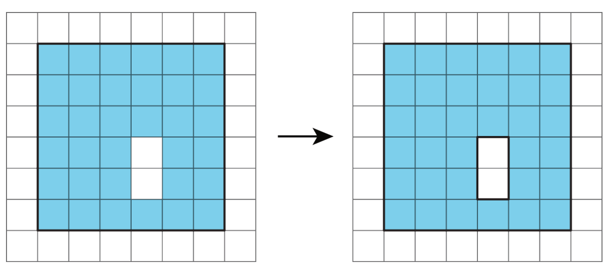
- Decompose it into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.



- Decompose it and rearrange the pieces into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.

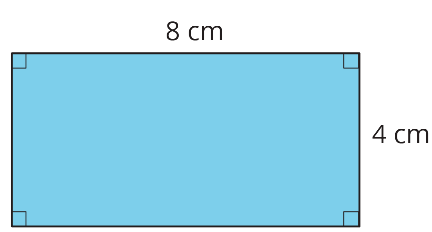


- Consider it as a shape with a missing piece; calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.



The area of a figure is always measured in square units. When both side lengths of a rectangle are given in centimeters, then the area is given in square centimeters.

The area of this rectangle is 32 square centimeters.



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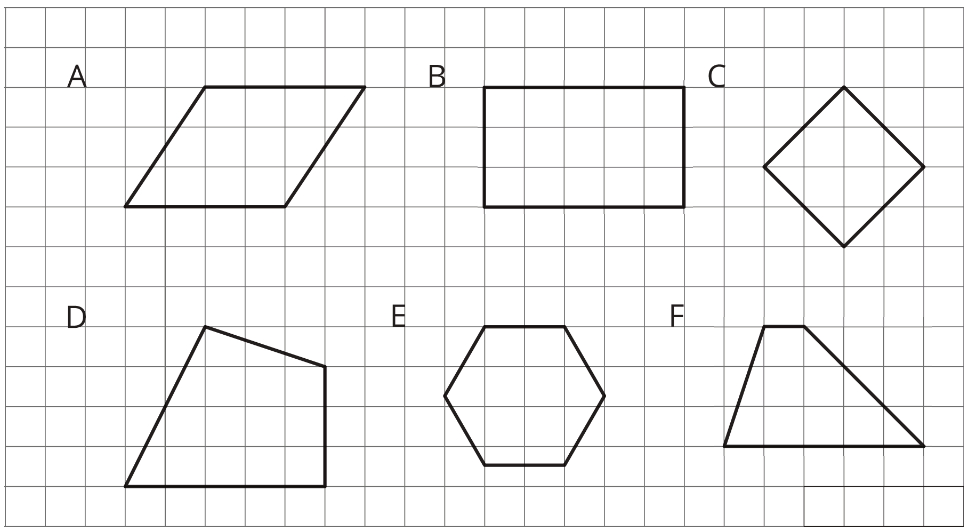
Unit 1, Lesson 4: Parallelograms

Let's investigate the features and area of parallelograms.

4.1: Features of a Parallelogram

Figures A, B, and C are **parallelograms**. Figures D, E, and F are *not* parallelograms.



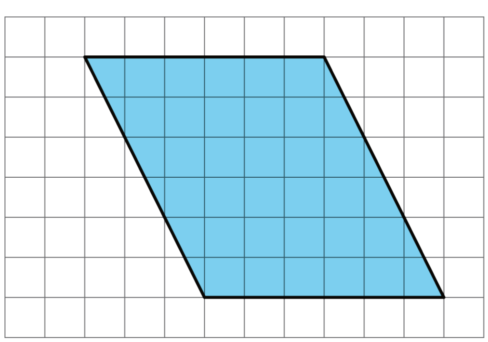
Study the examples and non-examples. What do you notice about:

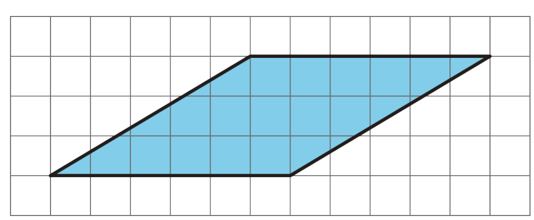
1. the number of sides that a parallelogram has?
2. opposite sides of a parallelogram?
3. opposite angles of a parallelogram?

4.2: Area of a Parallelogram

m.openup.org/1/6-1-4-2

Find the area of each parallelogram. Show your reasoning.





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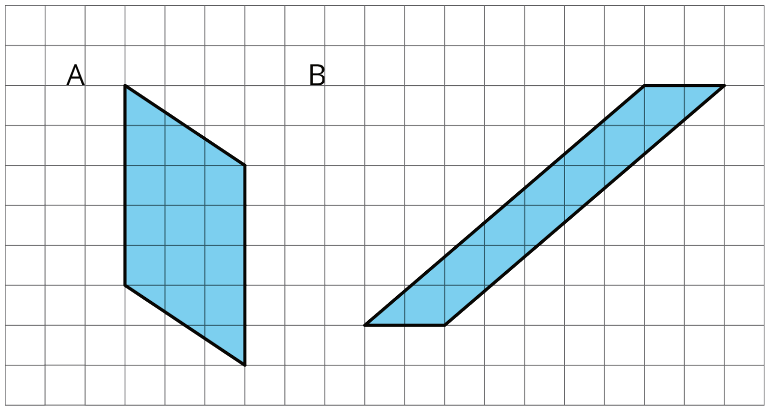
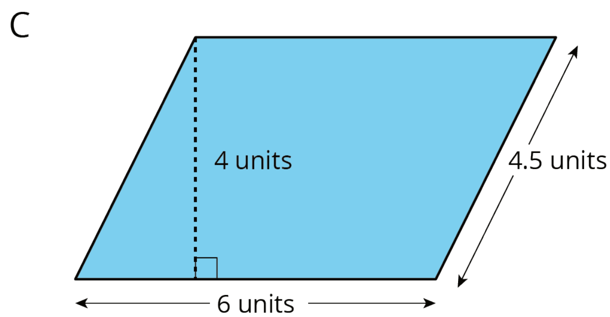
4.3: Lots of Parallelograms

Find the area of the following parallelograms. Show your reasoning.

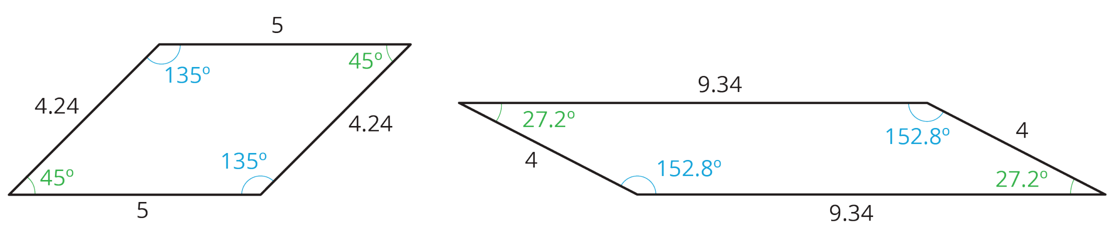




Lesson 4 Summary

A **parallelogram** is a quadrilateral (it has four sides). The opposite sides of a parallelogram are parallel. It is also true that:

- The opposite sides of a parallelogram have equal length.
- The opposite angles of a parallelogram have equal measure.



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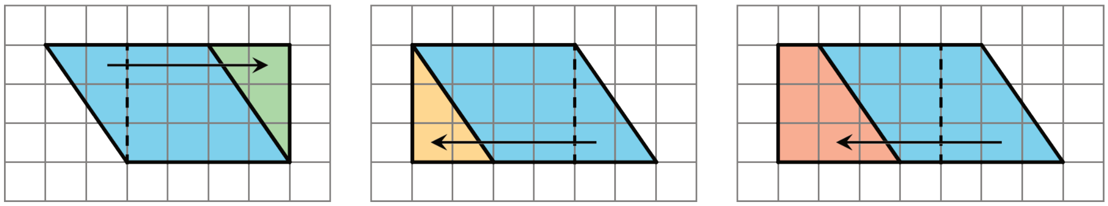
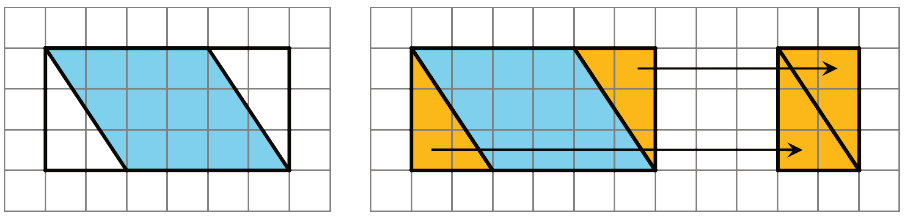
There are several strategies for finding the area of a **parallelogram**.

- We can decompose and rearrange a parallelogram to form a rectangle. Here are three ways:

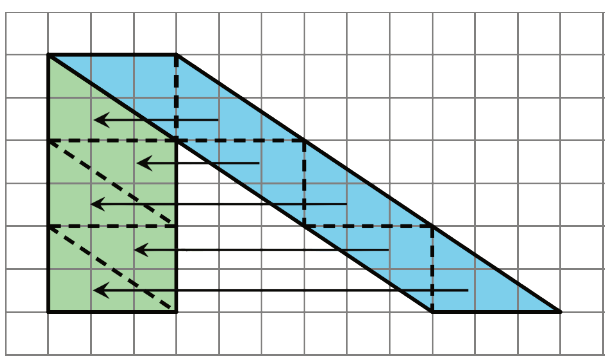


- We can enclose the parallelogram and then subtract the area of the two triangles in the corner.



Both of these ways will work for any parallelogram.

For some parallelograms, however, the process of decomposing and rearranging requires a lot more steps than if we enclose the parallelogram with a rectangle and subtract the combined area of the two triangles in the corners. Here is an example.



Lesson 4 Glossary Terms

- parallelogram

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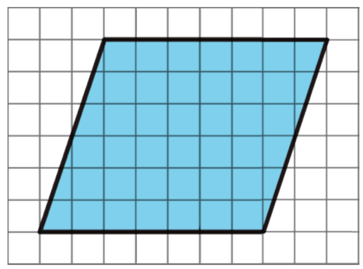
Unit 1, Lesson 5: Bases and Heights of Parallelograms

Let's investigate the area of parallelograms some more.

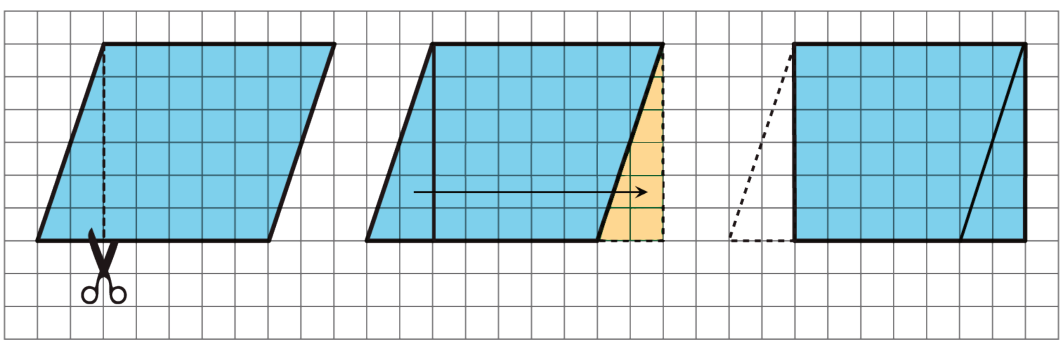
5.1: A Parallelogram and Its Rectangles

m.openup.org/1/6-1-5-1

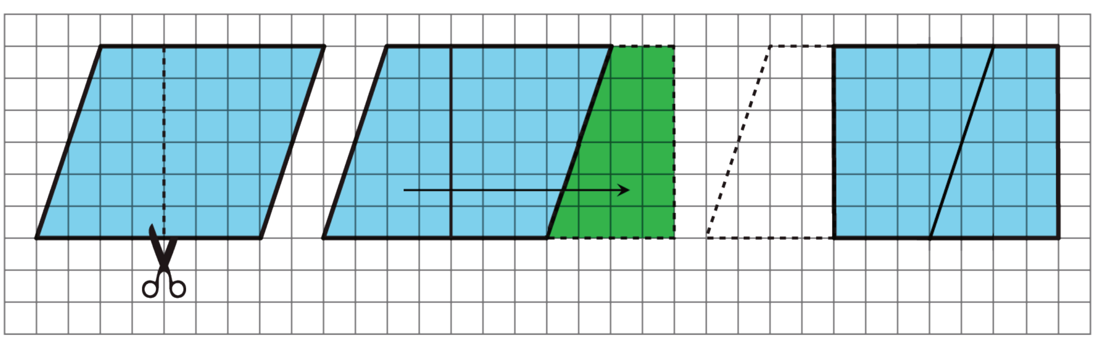
Elena and Tyler were finding the area of this parallelogram:



Here is how Elena did it:



Here is how Tyler did it:



How are the two strategies for finding the area of a parallelogram the same? How they are different?

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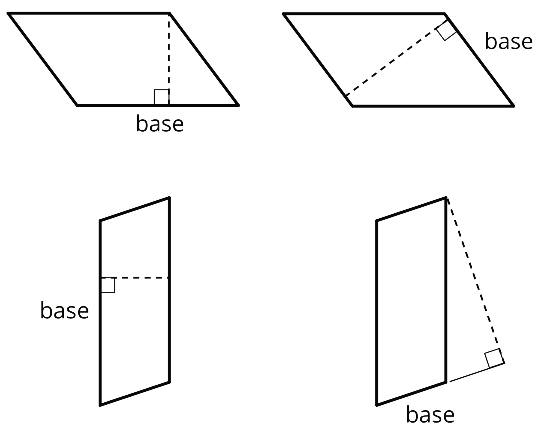
5.2: The Right Height?

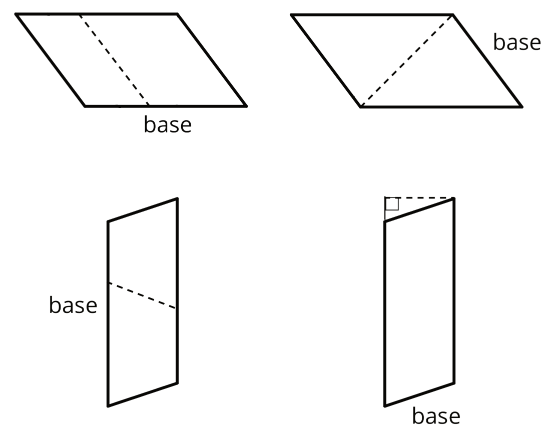
1. Each parallelogram has a side that is labeled “base.”

   Study the examples and non-examples of **bases** and **heights** of parallelograms. Then, answer the questions that follow.

   Examples: The dashed segment in each drawing represents the corresponding height for the given base.



   Non-examples: The dashed segment in each drawing does *not* represent the corresponding height for the given base.



   Select **all** statements that are true about bases and heights in a parallelogram.

   a. Only a horizontal side of a parallelogram can be a base.

   b. Any side of a parallelogram can be a base.

   c. A height can be drawn at any angle to the side chosen as the base.

   d. A base and its corresponding height must be perpendicular to each other.

   e. A height can only be drawn inside a parallelogram.

   f. A height can be drawn outside of the parallelogram, as long as it is drawn at a 90-degree angle to the base.

   g. A base cannot be extended to meet a height.

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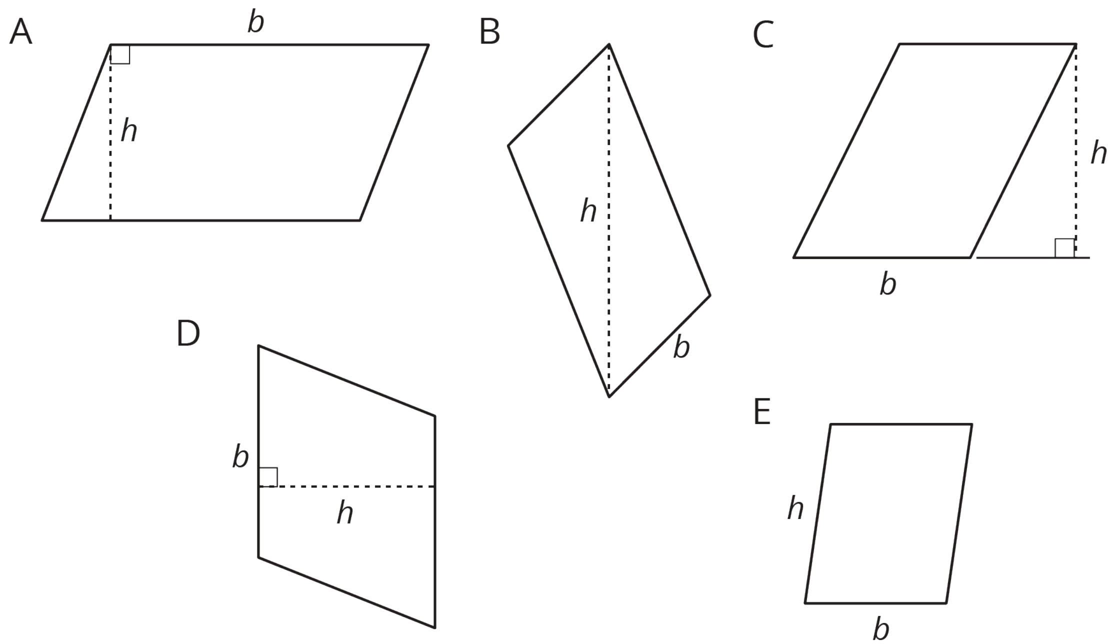
2. Five students labeled a base b and a corresponding height h for each of these parallelograms. Are all drawings correctly labeled? Explain how you know.



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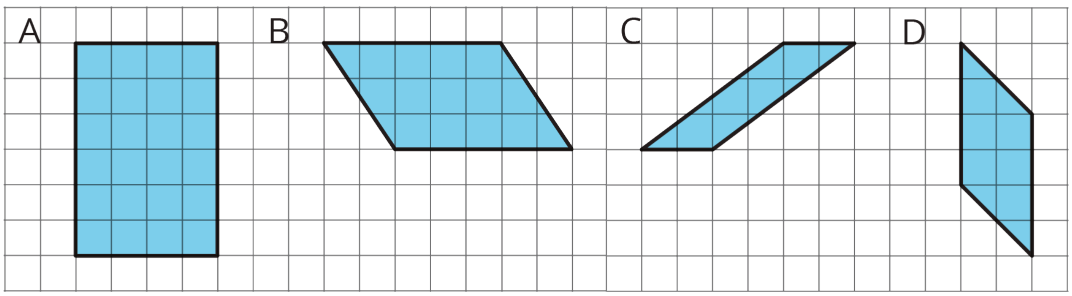
5.3: Finding the Formula for Area of Parallelograms

For each parallelogram:

- Identify a base and a corresponding height, and record their lengths in the table that follows.
- Find the area and record it in the right-most column.

In the last row, write an expression using b and h for the area of any parallelogram.



parallelogram	base (units)	height (units)	area (sq units)
A			
B			
C			
D			
any parallelogram	b	h	

Are you ready for more?

1. What happens to the area of a parallelogram if the height doubles but the base is unchanged? If the height triples? If the height is 100 times the original?
2. What happens to the area if both the base and the height double? Both triple? Both are 100 times their original lengths?

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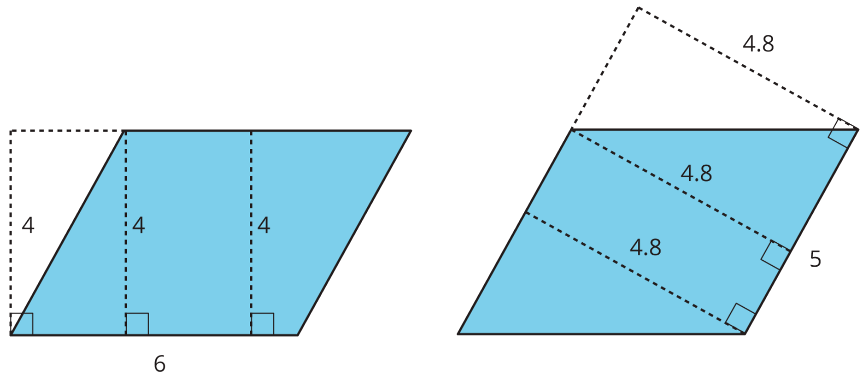
Lesson 5 Summary

- We can choose any of the four sides of a parallelogram as the **base**. Both the side (the segment) and its length (the measurement) are called the base.

- If we draw any perpendicular segment from a point on the base to the opposite side of the parallelogram, that segment will always have the same length. We call that value the **height**. There are infinitely many line segments that can represent the height!



Here are two copies of the same parallelogram. On the left, the side that is the base is 6 units long. Its corresponding height is 4 units. On the right, the side that is the base is 5 units long. Its height is 4.8 units. For both, three different segments are shown to represent the height. We could draw in many more!

No matter which side is chosen as the base, the area of the parallelogram is the product of that base and its corresponding height. We can check it:

$$4 \times 6 = 24$$

and

$$4.8 \times 5 = 24$$

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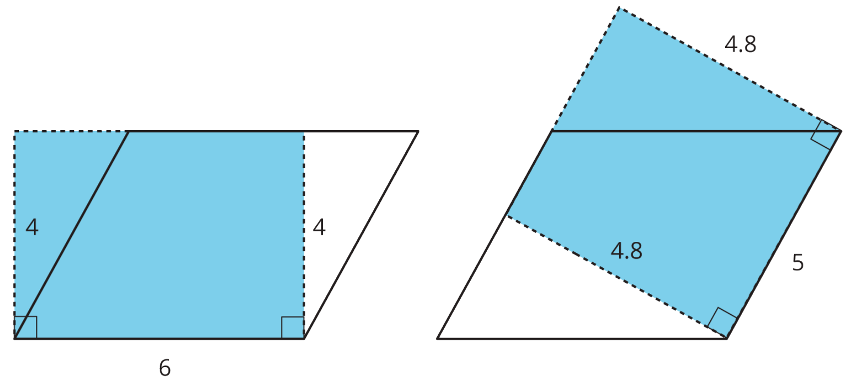
We can see why this is true by decomposing and rearranging the parallelograms into rectangles.



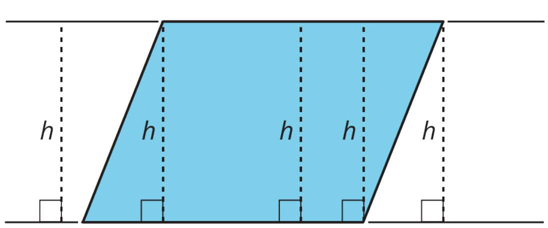
Notice that the side lengths of each rectangle are the base and height of the parallelogram. Even though the two rectangles have different side lengths, the products of the side lengths are equal, so they have the same area! And both rectangles have the same area as the parallelogram.

We often use letters to stand for numbers. If b is base of a parallelogram (in units), and h is the corresponding height (in units), then the area of the parallelogram (in square units) is the product of these two numbers.

$$b \cdot h$$

Notice that we write the multiplication symbol with a small dot instead of a $\times$ symbol. This is so that we don't get confused about whether $\times$ means multiply, or whether the letter x is standing in for a number.

In high school, you will be able to prove that a perpendicular segment from a point on one side of a parallelogram to the opposite side will always have the same length.



You can see this most easily when you draw a parallelogram on graph paper. For now, we will just use this as a fact.

Lesson 5 Glossary Terms

- base/height of a parallelogram

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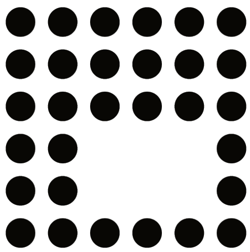
Unit 1, Lesson 6: Area of Parallelograms

Let's practice finding the area of parallelograms.

6.1: Missing Dots



How many dots are in the image?

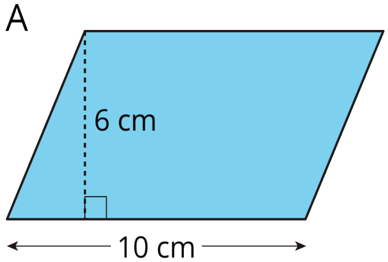
How do you see them?

6.2: More Areas of Parallelograms

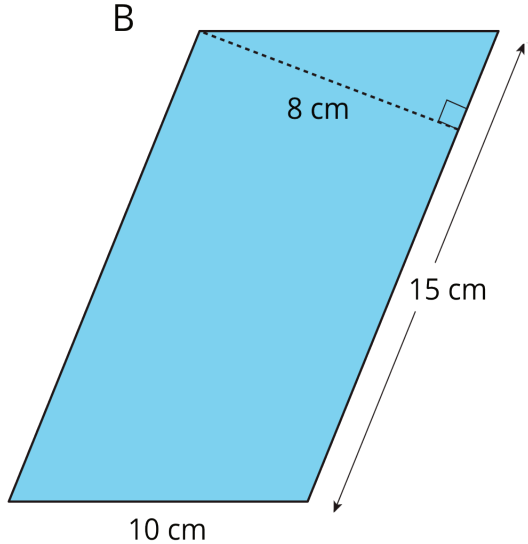
m.openup.org/1/6-1-6-2

1. Find the area of each parallelogram. Show your reasoning.

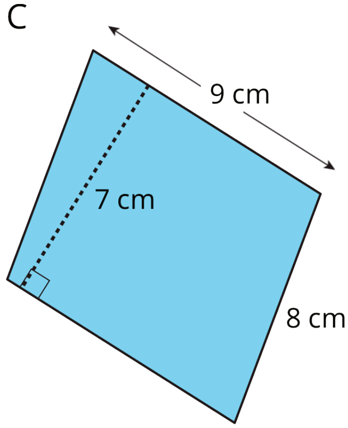
A



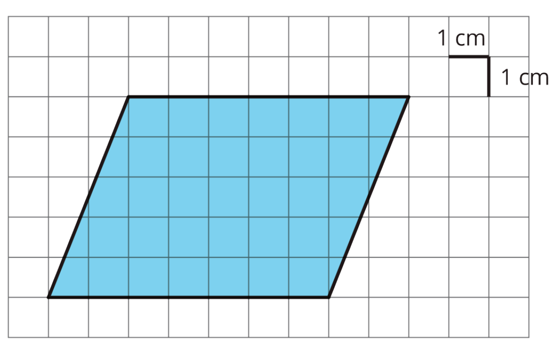
B



C



D



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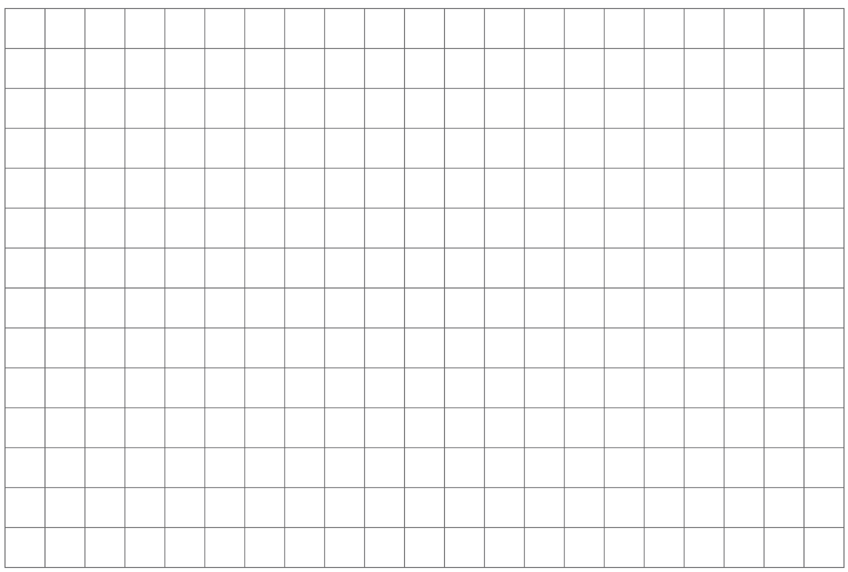
2. In Parallelogram B of the first question, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

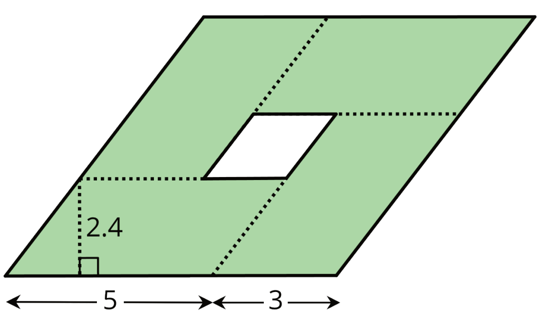
3. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

   On the grid, draw two parallelograms that could be P and Q.



Are you ready for more?

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.



What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

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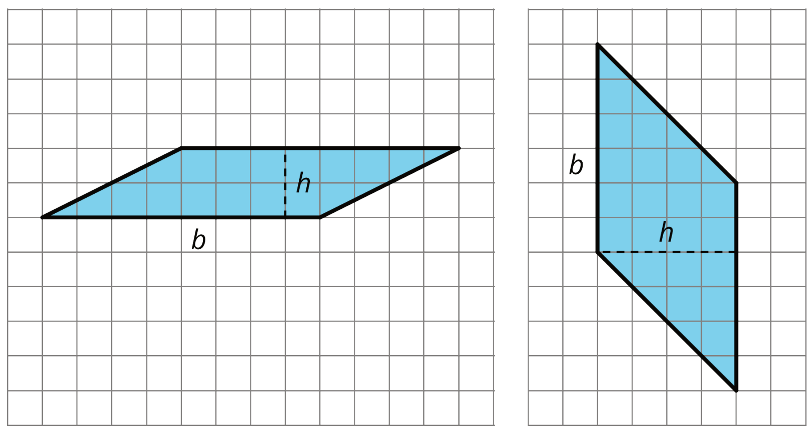
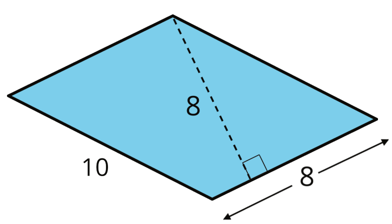
Lesson 6 Summary

Any pair of base and corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily identified than others.

- When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.

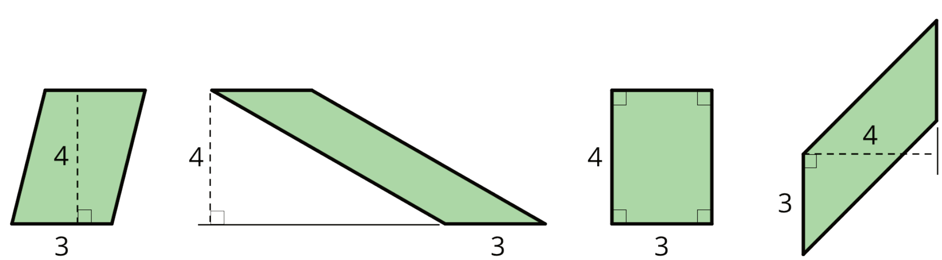


- When a parallelogram is *not* drawn on a grid, we can still find its area if a base and a corresponding height are known.



In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

Regardless of their shape, parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are some parallelograms with the same pair of base-height measurements.



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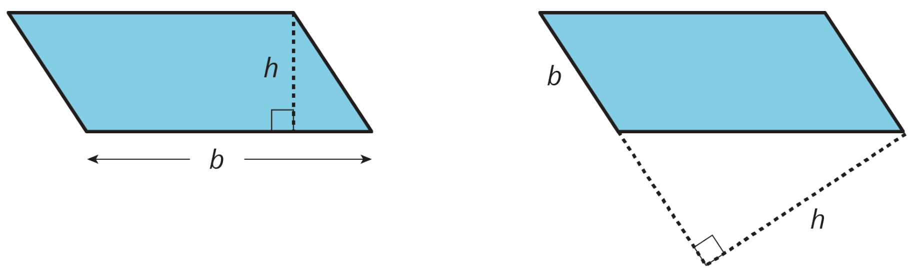
Unit 1, Lesson 7: From Parallelograms to Triangles

Let's compare parallelograms and triangles.

7.1: Same Parallelograms, Different Bases

Here are two copies of a parallelogram. Each copy has one side labeled as the base b and a segment drawn for its corresponding height and labeled h.



1. The base of the parallelogram on the left is 2.4 centimeters; its corresponding height is 1 centimeter. Find its area in square centimeters.

2. The height of the parallelogram on the right is 2 centimeters. How long is the base of that parallelogram? Explain your reasoning.

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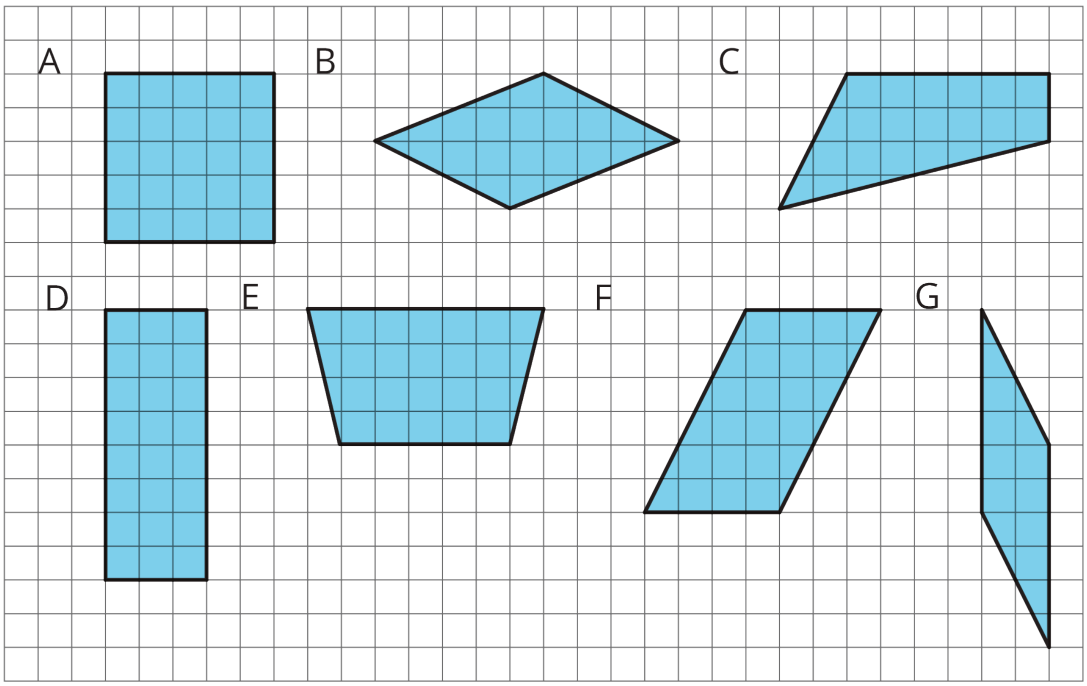
7.2: A Tale of Two Triangles (Part 1)

m.openup.org/1/6-1-7-2

Two polygons are identical if they match up exactly when placed one on top of the other.

1. Draw *one* line to decompose each of the following polygons into two identical triangles, if possible. Use a straightedge to draw your line.



2. Which quadrilaterals can be decomposed into two identical triangles?

   Pause here for a small-group discussion.

3. Study the quadrilaterals that can, in fact, be decomposed into two identical triangles. What do you notice about them? Write a couple of observations about what these quadrilaterals have in common.

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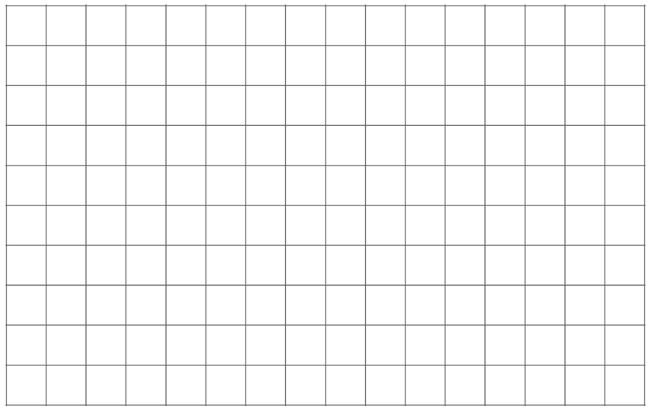
Are you ready for more?

On the grid, draw some other types of quadrilaterals that are not already shown. Try to decompose them into two identical triangles. Can you do it?



Come up with a rule about what must be true about a quadrilateral for it to be decomposed into two identical triangles.

7.3: A Tale of Two Triangles (Part 2)

m.openup.org/1/6-1-7-3

Your teacher will give your group several pairs of triangles. Each group member should take 1–2 pairs.

1. a. Which pair(s) of triangles do you have?

   b. Can each pair be composed into a rectangle? A parallelogram?

2. Discuss with your group your responses to the first question. Then, complete each of the following statements with *all*, *some*, or *none*. Sketch 1–2 examples to illustrate each completed statement.

   a. ________________ of these pairs of identical triangles can be composed into a *rectangle*.

   b. ________________ of these pairs of identical triangles can be composed into a *parallelogram*.

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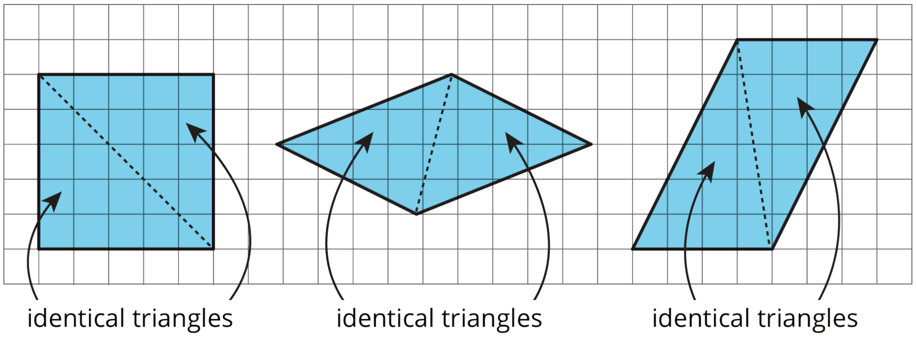
Lesson 7 Summary

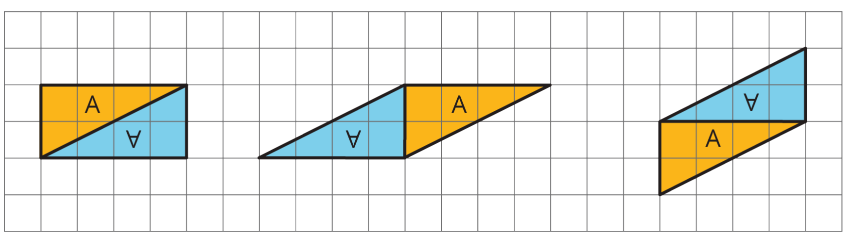
A parallelogram can always be decomposed into two identical triangles by a segment that connects opposite vertices.

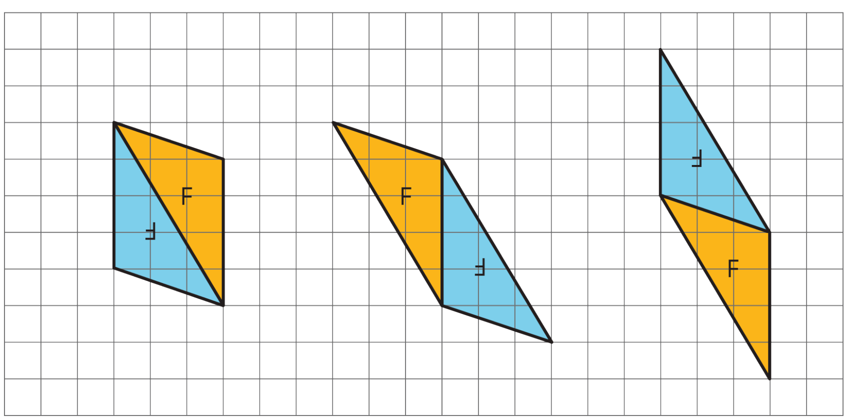


Going the other way around, two identical copies of a triangle can always be arranged to form a parallelogram, regardless of the type of triangle being used.

To produce a parallelogram, we can join a triangle and its copy along any of the three sides, so the same pair of triangles can make different parallelograms.

Here are examples of how two copies of both Triangle A and Triangle F can be composed into three different parallelograms.





This special relationship between triangles and parallelograms can help us reason about the area of any triangle.

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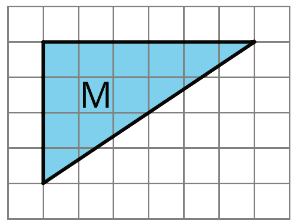
Unit 1, Lesson 8: Area of Triangles

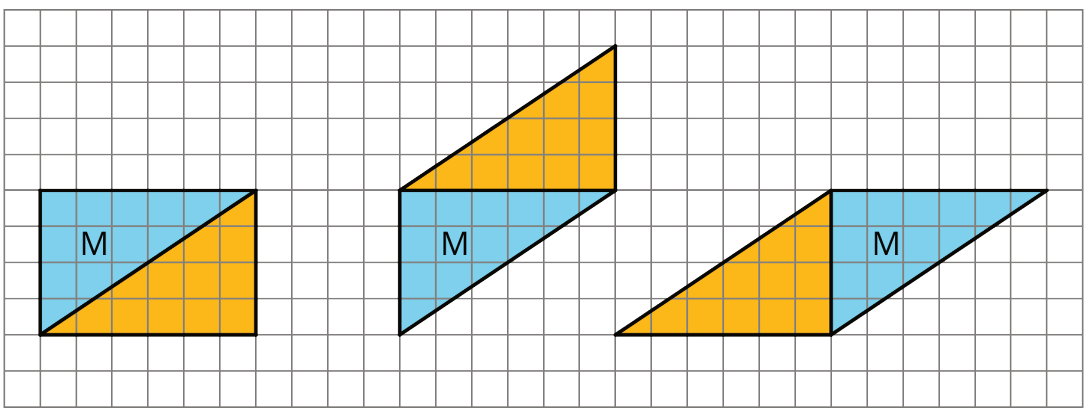
Let’s use what we know about parallelograms to find the area of triangles.

8.1: Composing Parallelograms

Here is Triangle M.



Han made a copy of Triangle M and composed three different parallelograms using the original M and the copy, as shown here.

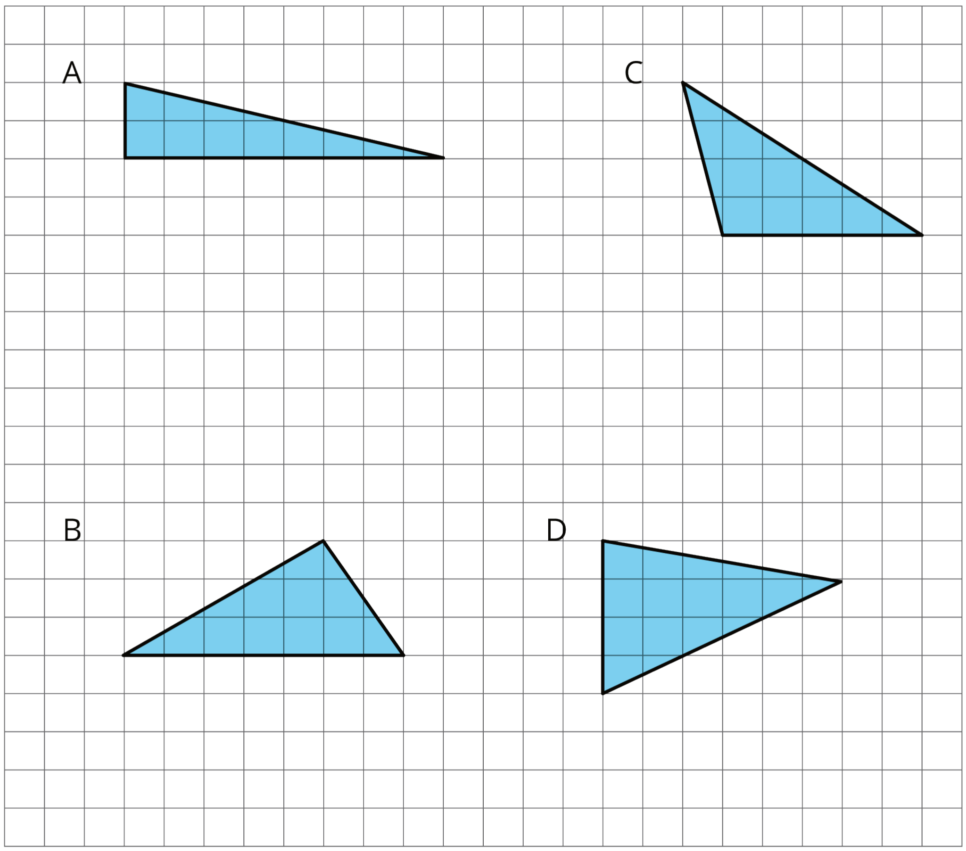


1. For each parallelogram Han composed, identify a base and a corresponding height, and write the measurements on the drawing.

2. Find the area of each parallelogram Han composed. Show your reasoning.

8.2: More Triangles

Find the areas of at least two of the triangles below. Show your reasoning.



8.3: Decomposing a Parallelogram

1. Your teacher will give you two copies of a parallelogram. Glue or tape *one* copy of your parallelogram here and find its area. Show your reasoning.

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2. Decompose the second copy of your parallelogram by cutting along the dotted lines. Take *only* the small triangle and the trapezoid, and rearrange these two pieces into a different parallelogram. Glue or tape the newly composed parallelogram on your paper.

3. Find the area of the new parallelogram you composed. Show your reasoning.

4. What do you notice about the relationship between the area of this new parallelogram and the original one?

5. How do you think the area of the large triangle compares to that of the new parallelogram: Is it larger, the same, or smaller? Why is that?

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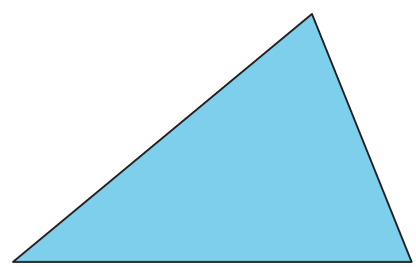
6. Glue or tape the remaining large triangle below. Use any part of the work above to help you find its area. Show your reasoning.

Are you ready for more?

Can you decompose this triangle and rearrange its parts to form a rectangle? Describe how it could be done.



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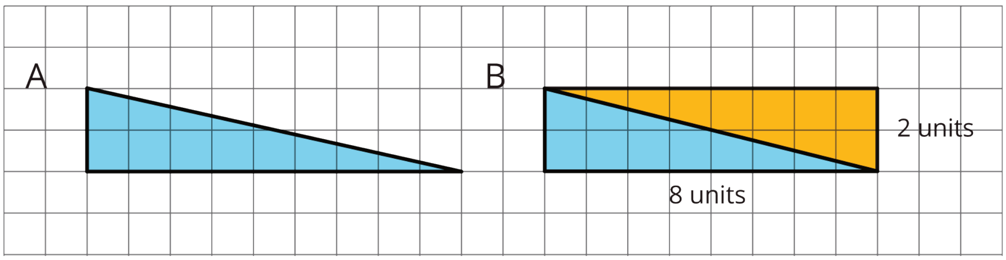
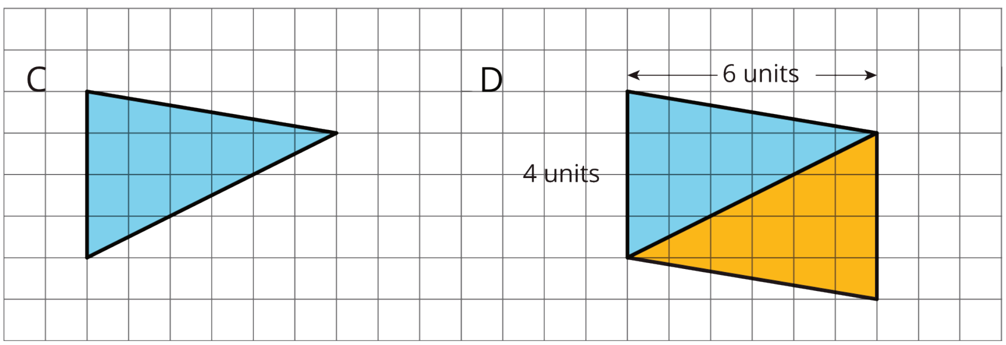
Lesson 8 Summary

We can reason about the area of a triangle by using what we know about parallelograms. Here are three general ways to do this:

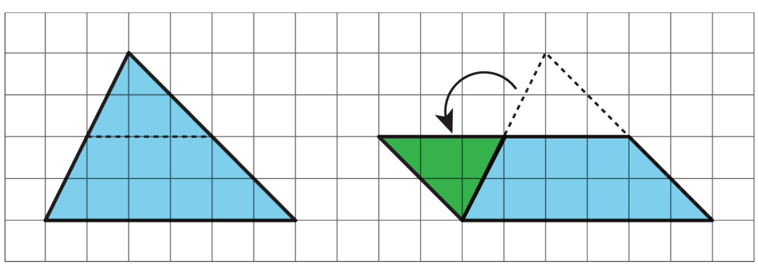
- Make a copy of the triangle and join the original and the copy along an edge to create a parallelogram. Because the two triangles have the same area, one copy of the triangle has half the area of that parallelogram.





The area of Parallelogram B is 16 square units because the base is 8 units and the height 2 units. The area of Triangle A is half of that, which is 8 square units. The area of Parallelogram D is 24 square units because the base is 4 units and the height 6 units. The area of Triangle C is half of that, which is 12 square units.

- Decompose the triangle into smaller pieces and compose them into a parallelogram.



In the new parallelogram, $b = 6$, $h = 2$, and $6 \cdot 2 = 12$, so its area is 12 square units. Because the original triangle and the parallelogram are composed of the same parts,

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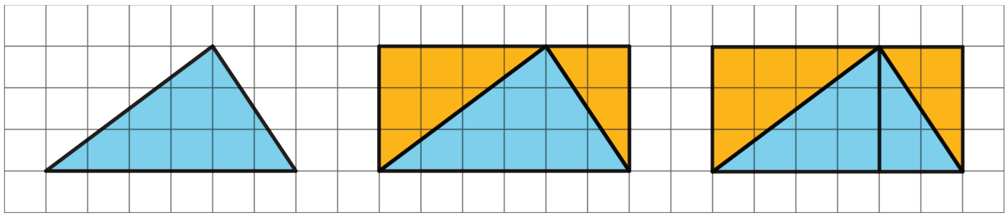
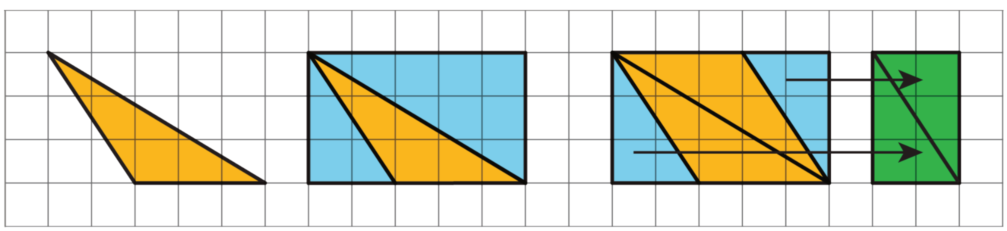
the area of the original triangle is also 12 square units.

- Draw a rectangle around the triangle. Sometimes the triangle has half of the area of the rectangle.



The large rectangle can be decomposed into smaller rectangles. The one on the left has area $4 \cdot 3$ or 12 square units; the one on the right has area $2 \cdot 3$ or 6 square units. The large triangle is also decomposed into two right triangles. Each of the right triangles is half of a smaller rectangle, so their areas are 6 square units and 3 square units. The large triangle has area 9 square units.

Sometimes, the triangle is half of what is left of the rectangle after removing two copies of the smaller right triangles.



The right triangles being removed can be composed into a small rectangle with area $(2 \cdot 3)$ square units. What is left is a parallelogram with area $5 \cdot 3 - 2 \cdot 3$, which equals $15 - 6$ or 9 square units. Notice that we can compose the same parallelogram with two copies of the original triangle! The original triangle is half of the parallelogram, so its area is $\frac{1}{2} \cdot 9$ or 4.5 square units.

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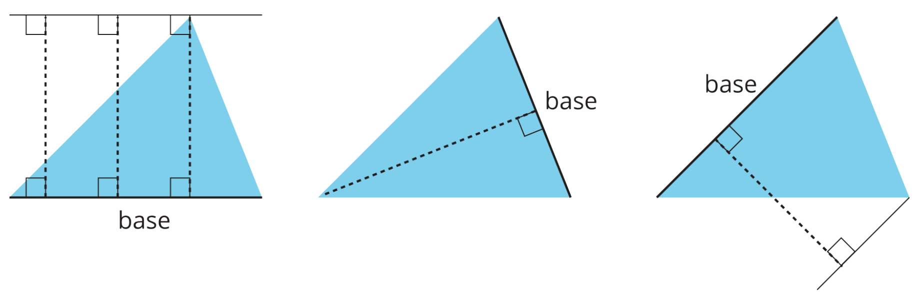
Unit 1, Lesson 9: Formula for the Area of a Triangle

Let’s write and use a formula to find the area of a triangle.

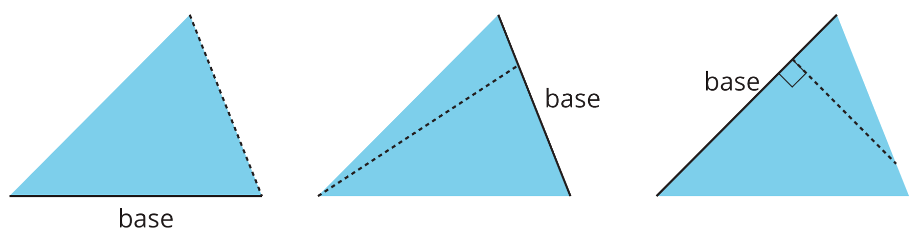
9.1: Bases and Heights of a Triangle

Study the examples and non-examples of **bases** and **heights** in a triangle. Answer the questions that follow.

- These dashed segments represent heights of the triangle.



- These dashed segments do *not* represent heights of the triangle.



Select **all** the statements that are true about bases and heights in a triangle.

1. Any side of a triangle can be a base.
2. There is only one possible height.
3. A height is always one of the sides of a triangle.
4. A height that corresponds to a base must be drawn at an acute angle to the base.
5. A height that corresponds to a base must be drawn at a right angle to the base.
6. Once we choose a base, there is only one segment that represents the corresponding height.
7. A segment representing a height must go through a vertex.

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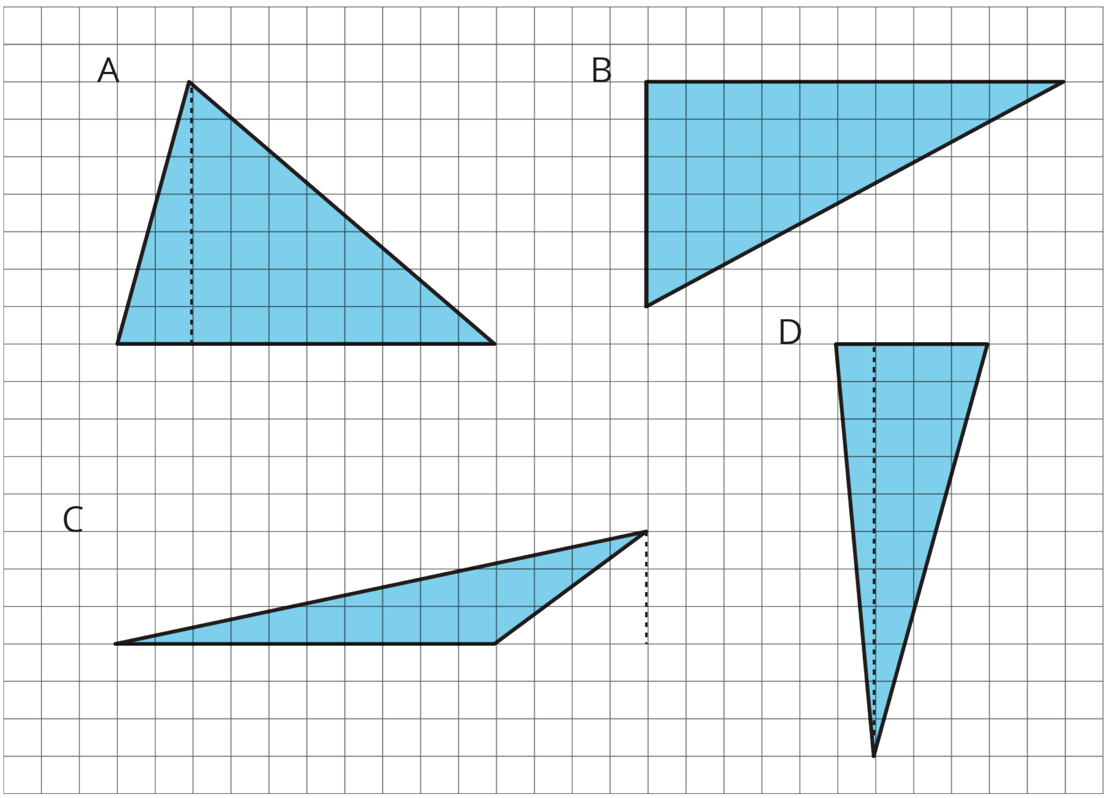
9.2: Finding the Formula for Area of a Triangle

- For each triangle, label a side that can be used as the base and a segment showing its corresponding height.
- Record the measurements for the base and height in the table, and find the area of the triangle. (The side length of each square on the grid is 1 unit.)
- In the last row, write an expression for the area of any triangle using b and h.



triangle	base (units)	height (units)	area (square units)
A			
B			
C			
D			
any triangle	b	h	

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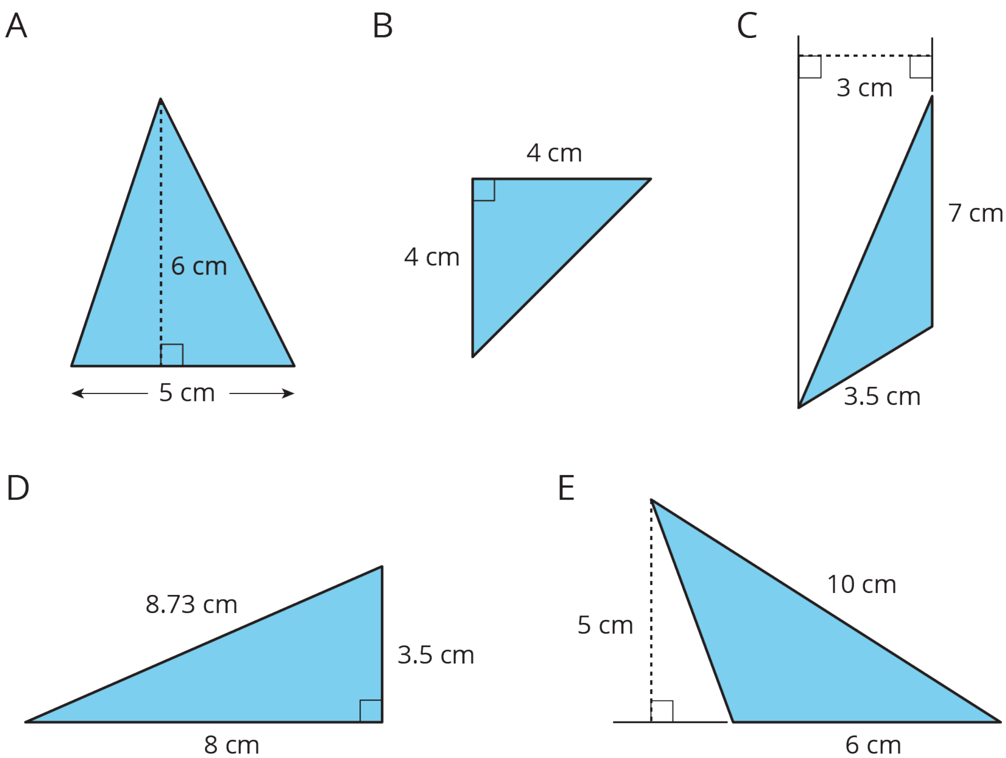
9.3: Applying the Formula for Area of Triangles

For each triangle, circle a base measurement that you can use to find the area of the triangle. Then, find the area of any *three* triangles. Show your reasoning.



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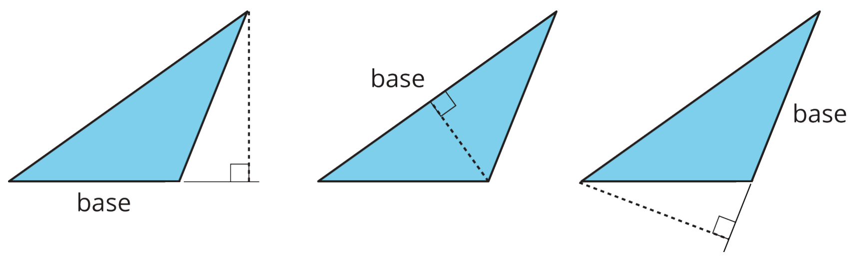
Lesson 9 Summary

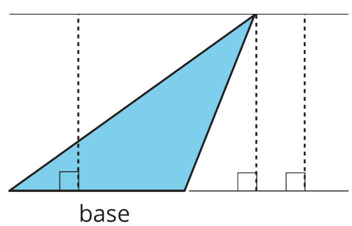
- We can choose any of the three sides of a triangle to call the **base**. The term “base” refers to both the side and its length (the measurement).

- The corresponding **height** is the length of a perpendicular segment from the base to the vertex opposite of it. The **opposite vertex** is the vertex that is *not* an endpoint of the base.

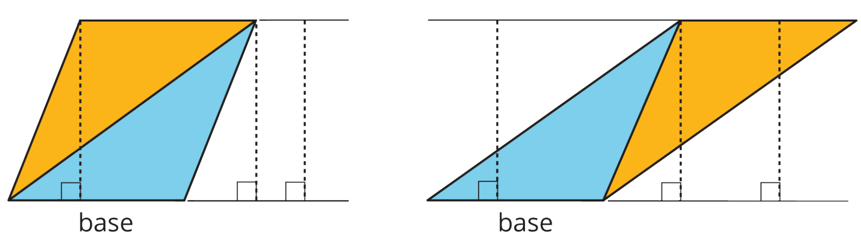
Here are three pairs of bases and heights for the same triangle. The dashed segments in the diagrams represent heights.



A segment showing a height must be drawn at a right angle to the base, but it can be drawn in more than one place. It does not have to go through the opposite vertex, as long as it connects the base and a line that is parallel to the base and goes through the opposite vertex, as shown here.



The base-height pairs in a triangle are closely related to those in a parallelogram. Recall that two copies of a triangle can be composed into one or more parallelograms. Each parallelogram shares at least one base with the triangle.



For any base that they share, the corresponding height is also shared, as shown by the dashed segments.

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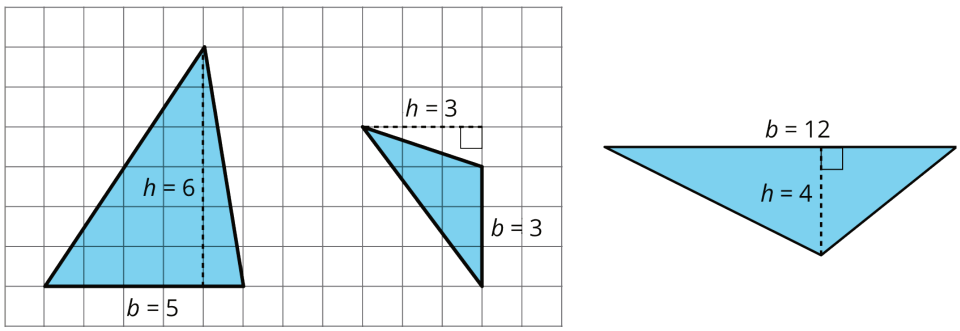
We can use the base-height measurements and our knowledge of parallelograms to find the area of any triangle.

- The formula for the area of a parallelogram with base b and height h is $b \cdot h$.
- A triangle takes up half of the area of a parallelogram with the same base and height. We can therefore express the area A of a triangle as:

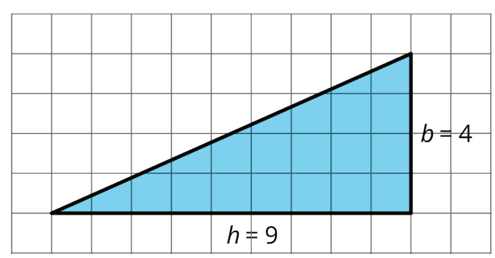
$$A = \frac{1}{2} \cdot b \cdot h$$



- The area of Triangle A is 15 square units because $\frac{1}{2} \cdot 5 \cdot 6 = 15$.
- The area of Triangle B is 4.5 square units because $\frac{1}{2} \cdot 3 \cdot 3 = 4.5$.
- The area of Triangle C is 24 square units because $\frac{1}{2} \cdot 12 \cdot 4 = 24$.

In each case, one side of the triangle is the base but neither of the other sides is the height. This is because the angle between them is not a right angle.

In right triangles, however, the two sides that are perpendicular can be a base and a height.



The area of this triangle is 18 square units whether we use 4 units or 9 units for the base.

Lesson 9 Glossary Terms

- opposite vertex
- base/height of a triangle

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Unit 1, Lesson 10: Bases and Heights of Triangles

Let's use different base-height pairs to find the area of a triangle.

10.1: An Area of 12

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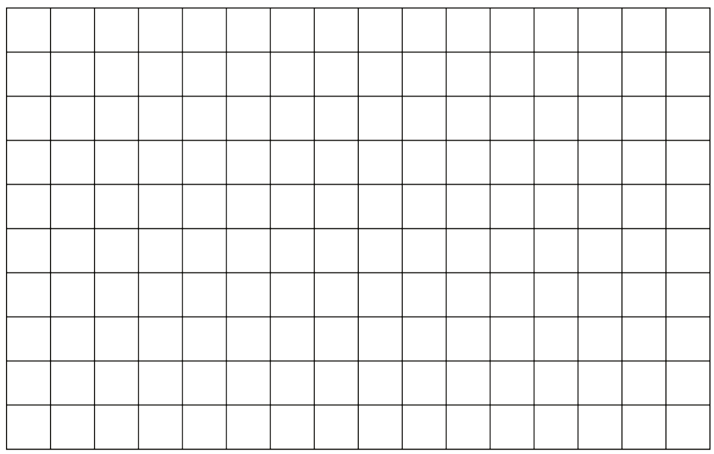
On the grid, draw a triangle with an area of 12 square units. Try to draw a non-right triangle. Be prepared to explain how you know the area of your triangle is 12 square units.



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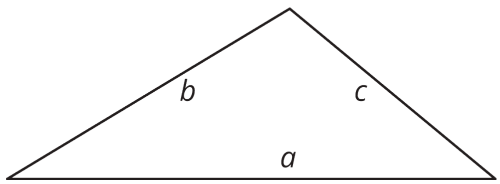
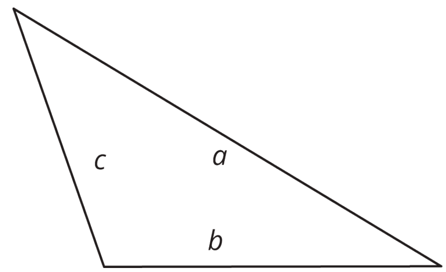
10.2: Hunting for Heights

1. Here are three copies of the same triangle. The triangle is rotated so that the side chosen as the base is at the bottom and is horizontal. Draw a height that corresponds to each base. Use an index card to help you.

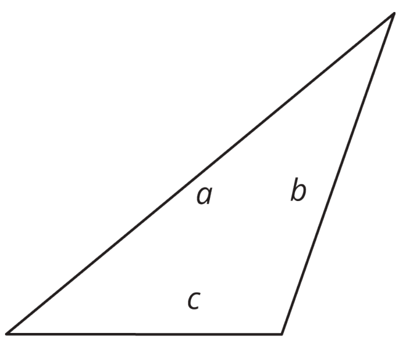
Side a as the base:



Side b as the base:



Side c as the base:



Pause for your teacher's instructions before moving to the next question.

2. Draw a line segment to show the height for the chosen base in each triangle.

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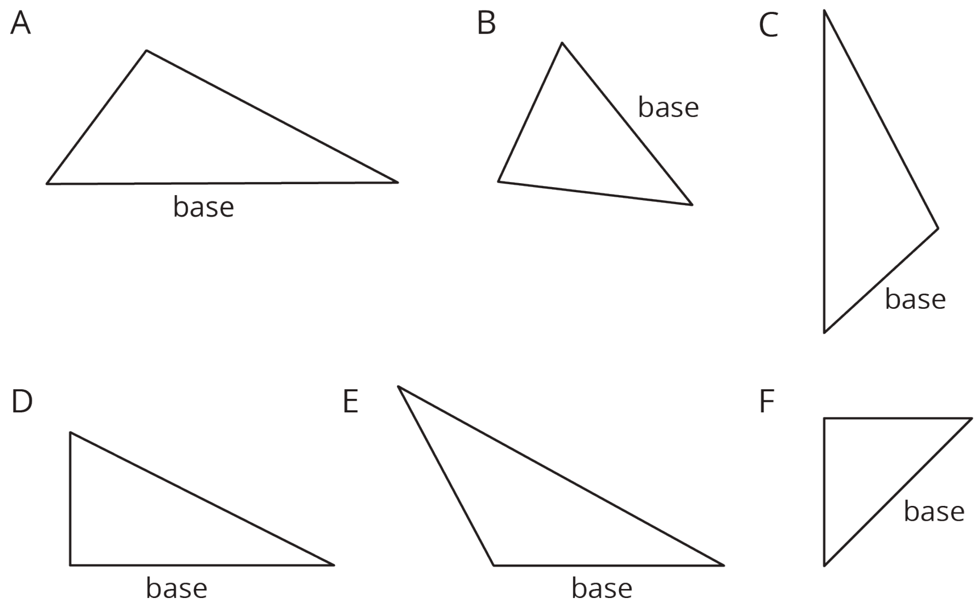

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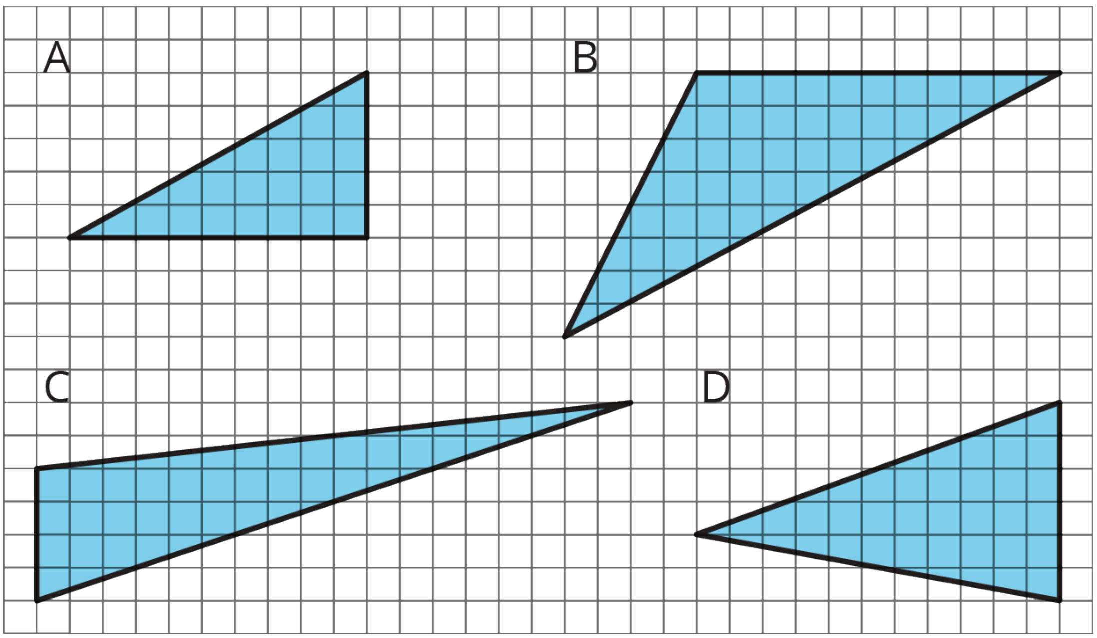
10.3: Some Bases Are Better Than Others

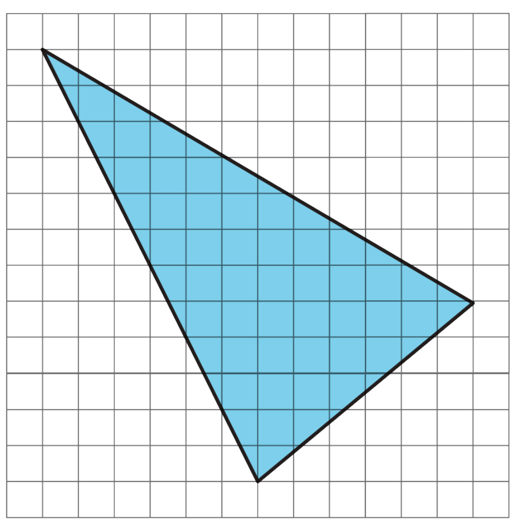
For each triangle, identify and label a base and height. If needed, draw a line segment to show the height.

Then, find the area of the triangle. Show your reasoning. (The side length of each square on the grid is 1 unit.)



Are you ready for more?

Find the area of the triangle below. Show your reasoning.



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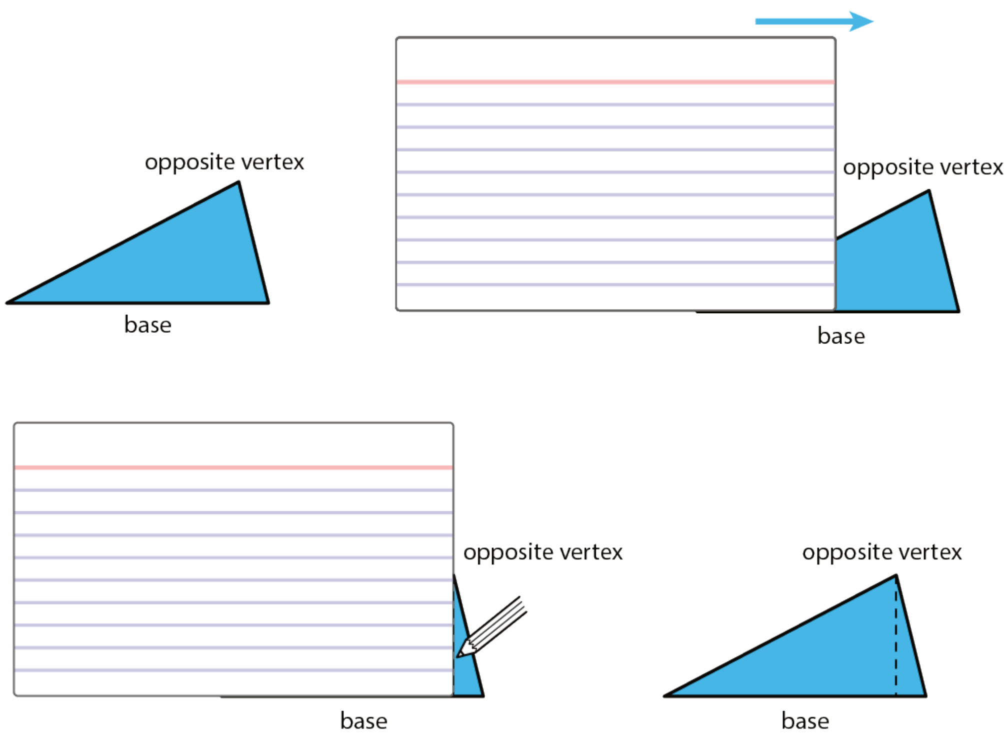
Lesson 10 Summary

A height of a triangle is a perpendicular segment between the side chosen as the base and the opposite vertex. We can use tools with right angles to help us draw height segments.

An index card (or any stiff paper with a right angle) is a handy tool for drawing a line that is perpendicular to another line.

1. Choose a side of a triangle as the base. Identify its opposite vertex.
2. Line up one edge of the index card with that base.
3. Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
4. Use the card edge to draw a line from the vertex to the base. That segment represents the height.



Sometimes we may need to extend the line of the base to identify the height, such as when finding the height of an obtuse triangle, or whenever the opposite vertex is not directly over the base. In these cases, the height segment is typically drawn *outside* of the triangle.

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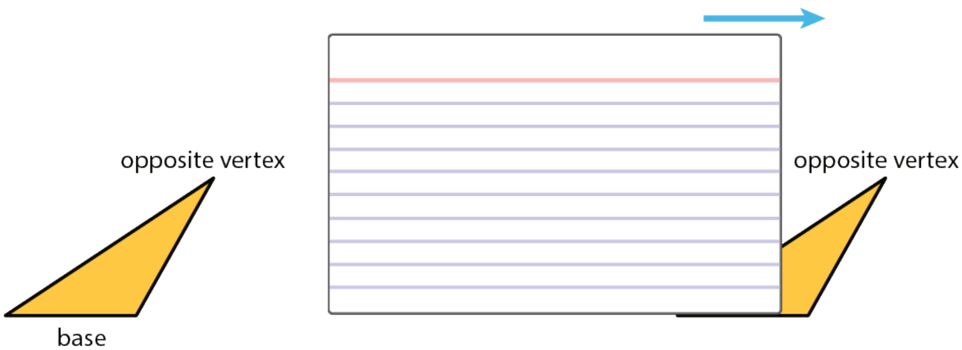
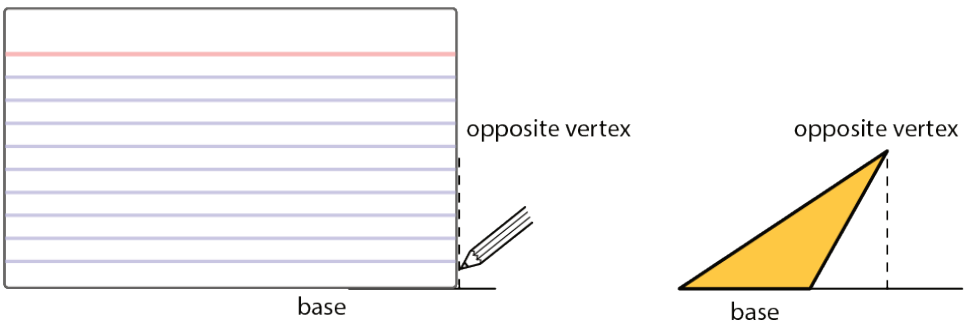
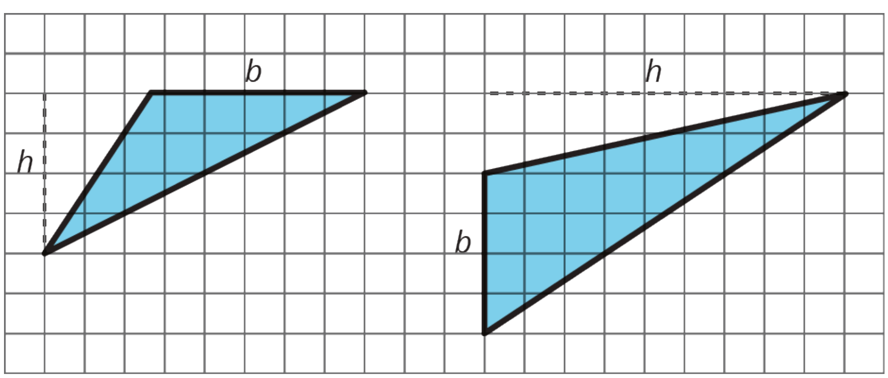




Even though any side of a triangle can be a base, some base-height pairs can be more easily determined than others, so it helps to choose strategically.

For example, when dealing with a right triangle, it often makes sense to use the two sides that make the right angle as the base and the height because one side is already perpendicular to the other.

If a triangle is on a grid and has a horizontal or a vertical side, you can use that side as a base and use the grid to find the height, as in these examples:



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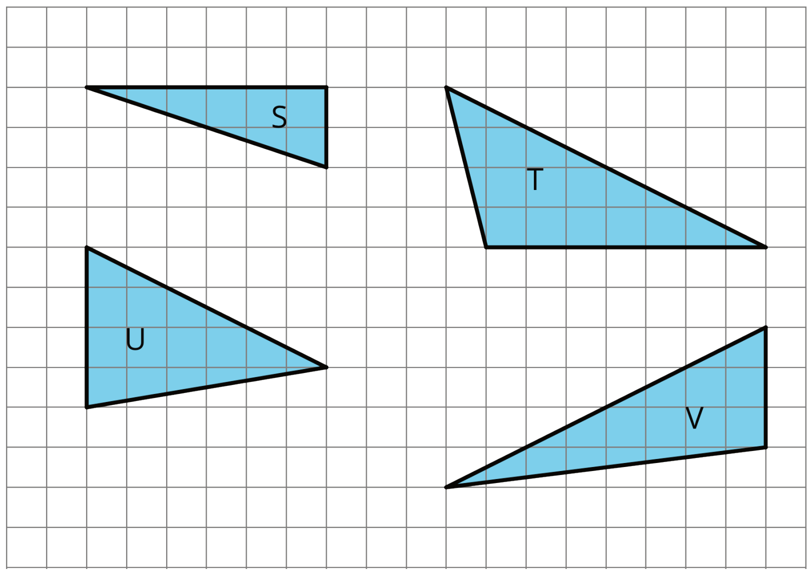
Unit 1, Lesson 11: Polygons

Let’s investigate polygons and their areas.

11.1: Which One Doesn’t Belong: Bases and Heights

Which one doesn't belong?



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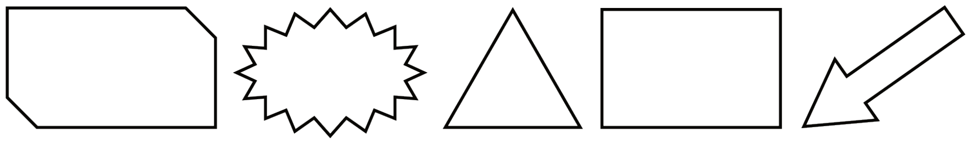
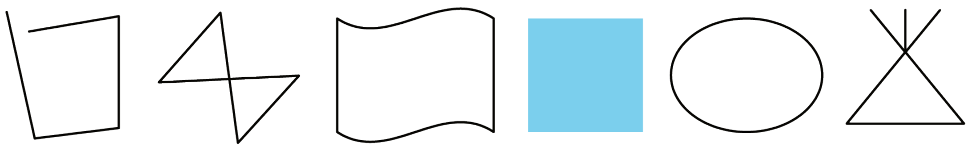
11.2: What Are Polygons?

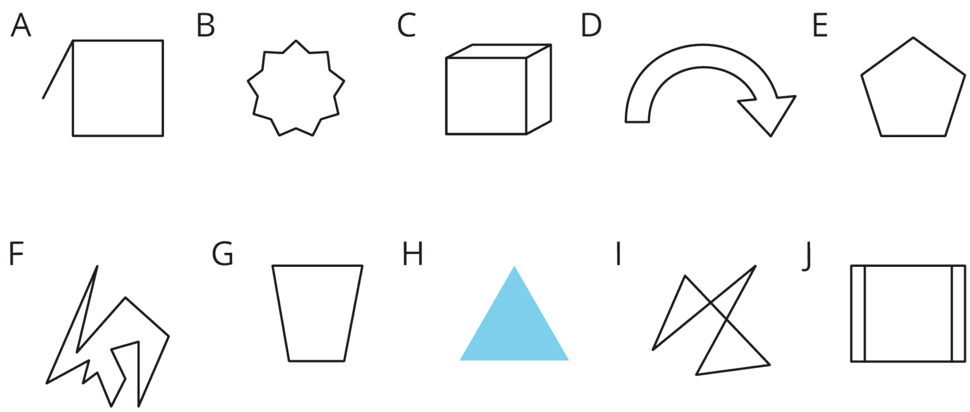
Here are five **polygons**:



Here are six figures that are *not* polygons:



1. Circle the figures that are polygons.



2. What do the figures you circled have in common? What characteristics helped you decide whether a figure was a polygon?

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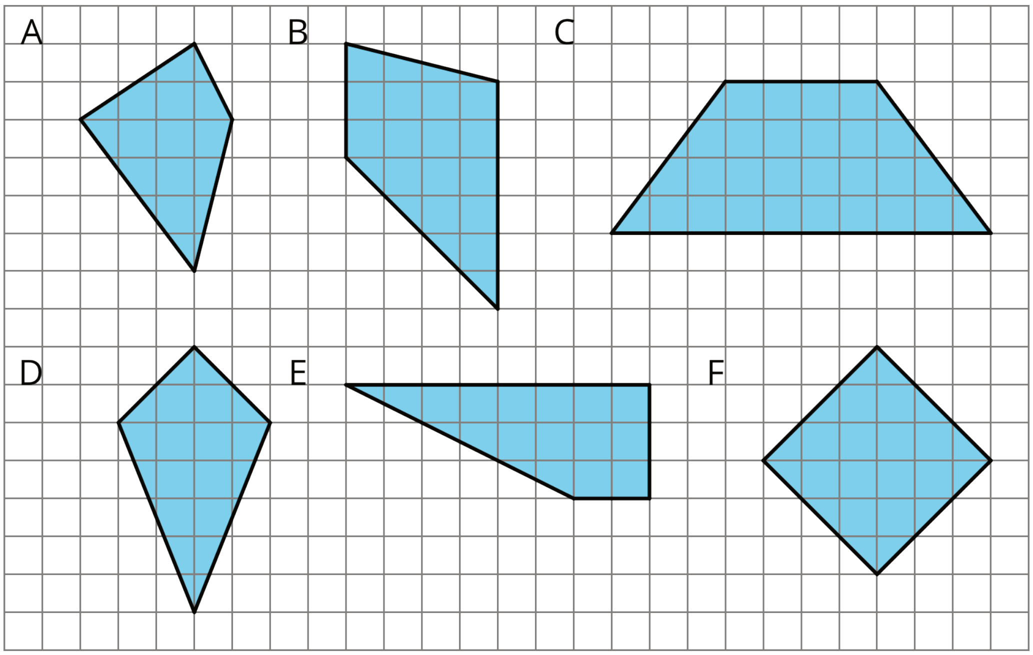
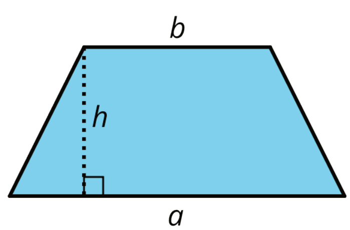
11.3: Quadrilateral Strategies

Find the area of *two* **quadrilaterals** of your choice. Show your reasoning.



Are you ready for more?

Here is a trapezoid. a and b represent the lengths of its bottom and top sides. The segment labeled h represents its height; it is perpendicular to both the top and bottom sides.



Apply area-reasoning strategies—decomposing, rearranging, duplicating, etc.—on the trapezoid so that you have one or more shapes with areas that you already know how to find.

Use the shapes to help you write a formula for the area of a trapezoid. Show your reasoning.

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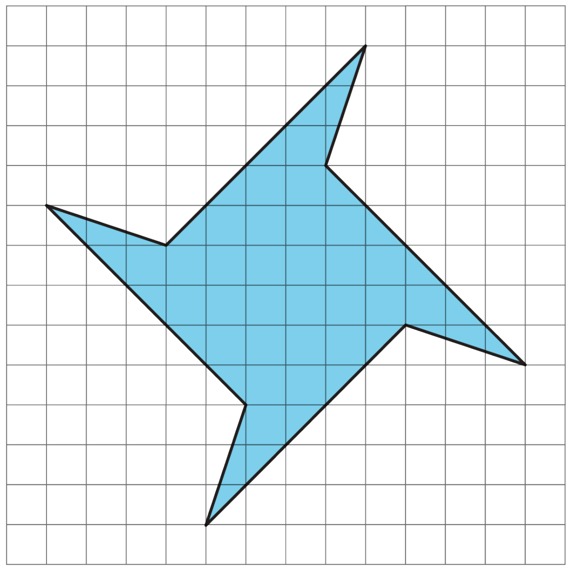
11.4: Pinwheel

Find the area of the shaded region in square units. Show your reasoning.



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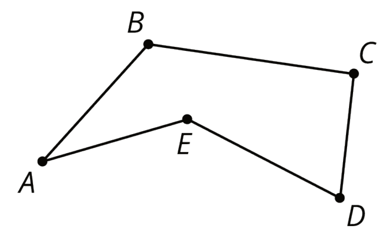
Lesson 11 Summary

A **polygon** is a two-dimensional figure composed of straight line segments.

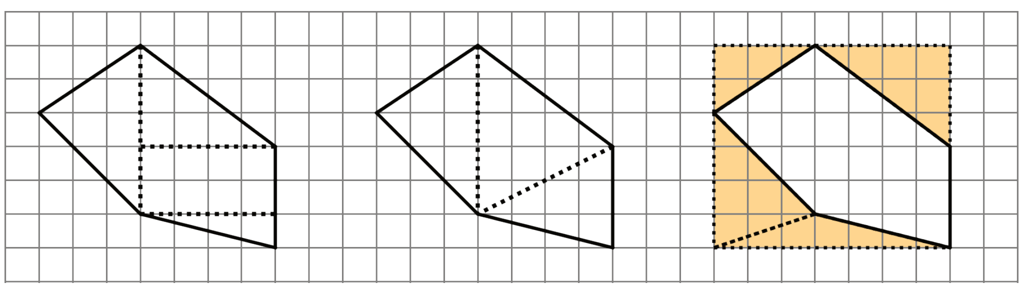
- Each end of a line segment connects to one other line segment. The point where two segments connect is a **vertex**. The plural of vertex is vertices.
- The segments are called the **edges** or **sides** of the polygon. The sides never cross each other. There are always an equal number of vertices and sides.

Here is a polygon with 5 sides. The vertices are labeled A, B, C, D, and E.



A polygon encloses a region. To find the area of a polygon is to find the area of the region inside it.

We can find the area of a polygon by decomposing the region inside it into triangles and rectangles.



The first two diagrams show the polygon decomposed into triangles and rectangles; the sum of their areas is the area of the polygon. The last diagram shows the polygon enclosed with a rectangle; subtracting the areas of the triangles from the area of the rectangle gives us the area of the polygon.

Lesson 11 Glossary Terms

- quadrilateral
- vertex (vertices)
- edge
- side

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- polygon

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Unit 1, Lesson 12: What is Surface Area?

Let’s cover the surfaces of some three-dimensional objects.

12.1: Covering the Cabinet (Part 1)

Your teacher will show you a video about a cabinet or some pictures of it.

Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom?

12.2: Covering the Cabinet (Part 2)

Earlier, you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?

2. Use the information you have to find the number of sticky notes to cover the cabinet. Show your reasoning.

Are you ready for more?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

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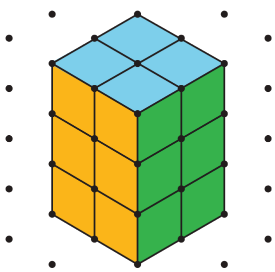
12.3: Building with Snap Cubes

m.openup.org/1/6-1-12-3

Here is a sketch of a rectangular prism built from 12 cubes:



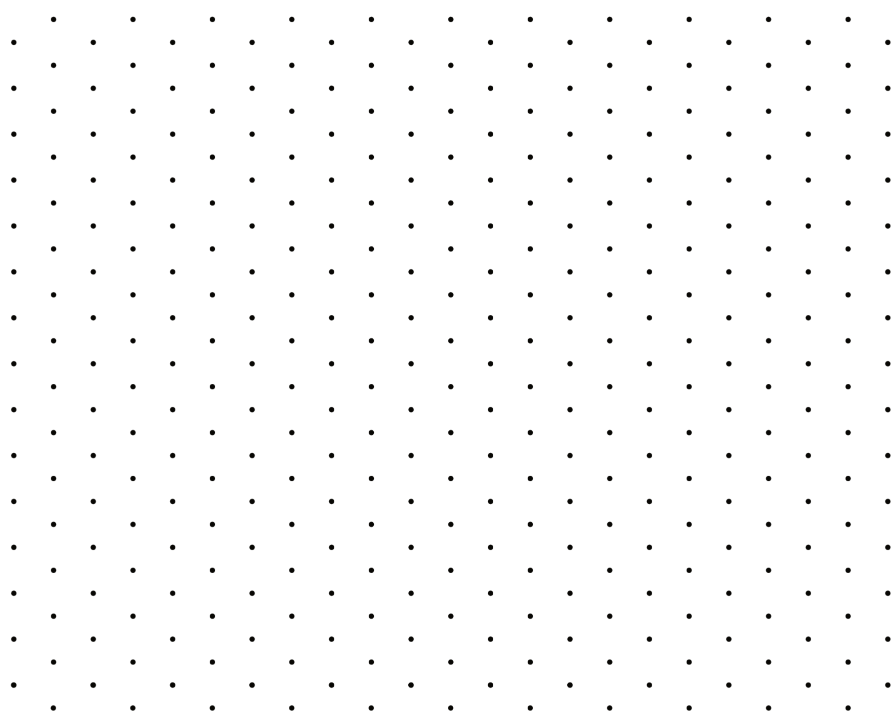
It has six **faces**, but you can only see three of them in the sketch. It has a **surface area** of 32 square units.

You have 12 snap cubes from your teacher. Use all of your snap cubes to build a different rectangular prism (with different edge lengths than shown in the prism here).

1. How many faces does your figure have?

2. What is the surface area of your figure in square units?

3. Draw your figure on isometric dot paper. Color each face a different color.



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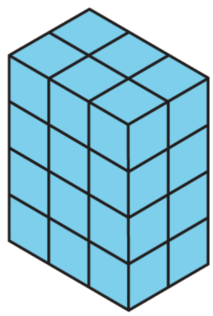
Lesson 12 Summary

- The **surface area** of a figure (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.
- If a three-dimensional figure has flat sides, the sides are called **faces**.
- The surface area is the total of the areas of the faces.

For example, a rectangular prism has six faces. The surface area of the prism is the total of the areas of the six rectangular faces.



So the surface area of a rectangular prism that has edge-lengths 2 cm, 3 cm, and 4 cm has a surface area of

$$(2 \cdot 3) + (2 \cdot 3) + (2 \cdot 4) + (2 \cdot 4) + (3 \cdot 4) + (3 \cdot 4)$$

or 52 square centimeters.

Lesson 12 Glossary Terms

- surface area
- face

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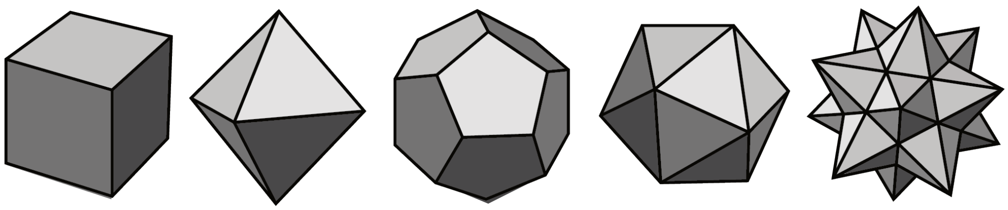
Unit 1, Lesson 13: Polyhedra

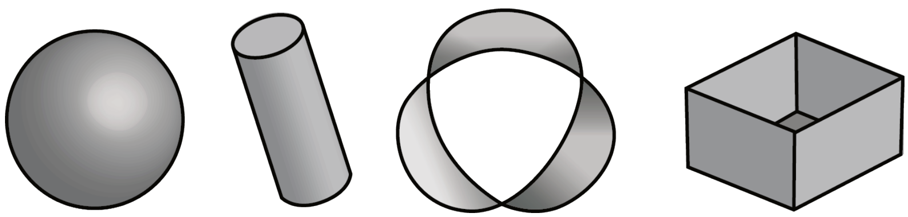
Let's investigate polyhedra.

13.1: What are Polyhedra?

Here are pictures that represent **polyhedra**:



Here are pictures that do *not* represent polyhedra:



1. Your teacher will give you some figures or objects. Sort them into polyhedra and non-polyhedra.

2. What features helped you distinguish the polyhedra from the other figures?

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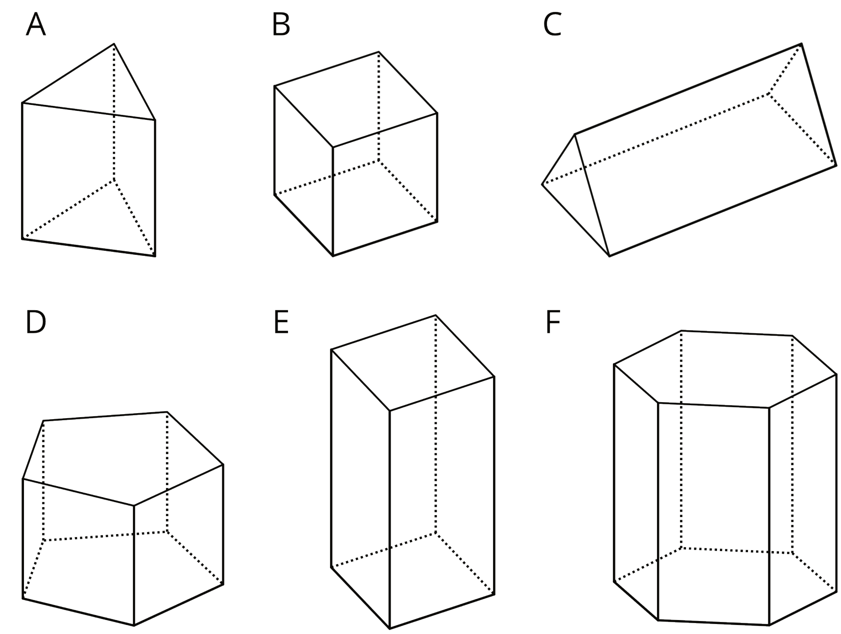
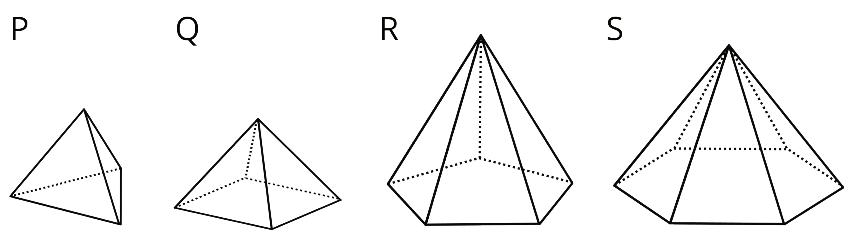
13.2: Prisms and Pyramids

1. Here are some polyhedra called **prisms**.



Here are some polyhedra called **pyramids**.



a. Look at the prisms. What are their characteristics or features?

b. Look at the pyramids. What are their characteristics or features?

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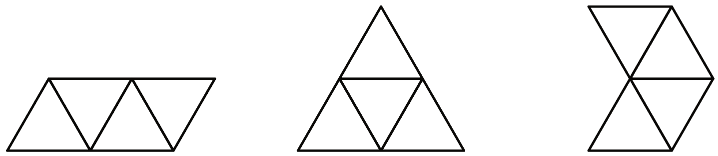
2. Which of the following **nets** can be folded into Pyramid P? Select all that apply.



3. Your teacher will give your group a set of polygons and assign a polyhedron.

    a. Decide which polygons are needed to compose your assigned polyhedron. List the polygons and how many of each are needed.

    b. Arrange the cut-outs into a net that, if taped and folded, can be assembled into the polyhedron. Sketch the net. If possible, find more than one way to arrange the polygons (show a different net for the same polyhedron).

Are you ready for more?

What is the smallest number of faces a polyhedron can possibly have? Explain how you know.

13.3: Assembling Polyhedra

1. Your teacher will give you the net of a polyhedron. Cut out the net, and fold it along the edges to assemble a polyhedron. Tape or glue the flaps so that there are no unjoined edges.

2. How many **vertices**, **edges**, and **faces** are in your polyhedron?

Lesson 13 Summary

A **polyhedron** is a three-dimensional figure composed of **faces.** Each face is a filled-in

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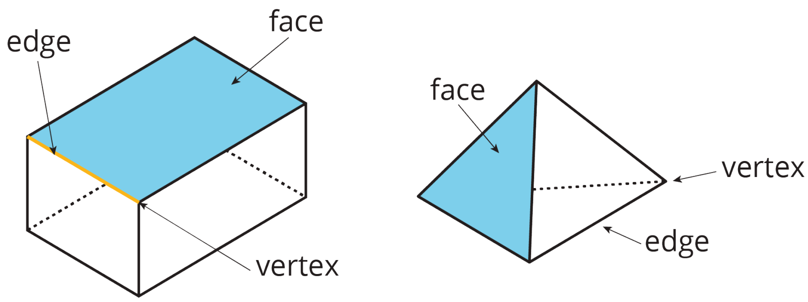
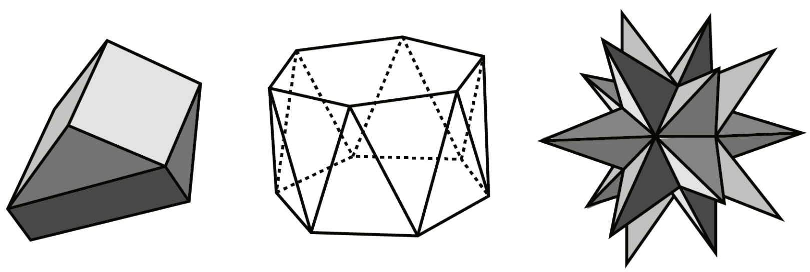
polygon and meets only one other face along a complete **edge**. The ends of the edges meet at points that are called **vertices**.



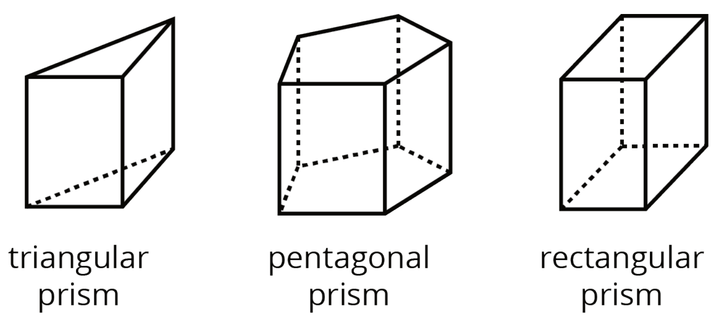
A **polyhedron** always encloses a three-dimensional region.

The plural of polyhedron is **polyhedra**. Here are some drawings of polyhedra:



A **prism** is a type of polyhedron with two identical faces that are parallel to each other and that are called *bases*. The bases are connected by a set of rectangles (or sometimes parallelograms).

A prism is named for the shape of its bases. For example, if the base is a pentagon, then it is called a “pentagonal prism.”



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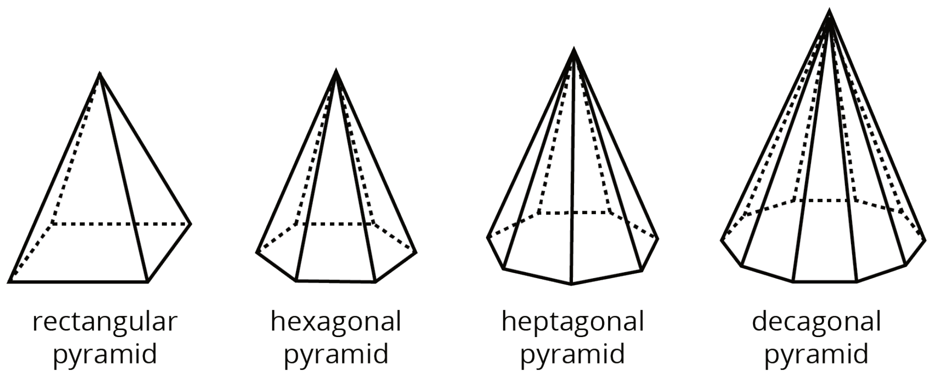
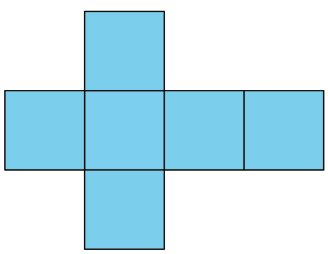
A **pyramid** is a type of polyhedron that has one special face called the base. All of the other faces are triangles that all meet at a single **vertex**.

A pyramid is named for the shape of its base. For example, if the base is a pentagon, then it is called a “pentagonal pyramid.”



A **net** is a two-dimensional representation of a polyhedron. It is composed of polygons that form the faces of a polyhedron.



A cube has 6 square faces, so its net is composed of six squares, as shown here.

A net can be cut out and folded to make a model of the polyhedron.

In a cube, every face shares its edges with 4 other squares. In a net of a cube, not all edges of the squares are joined with another edge. When the net is folded, however, each of these open edges will join another edge.

It takes practice to visualize the final polyhedron by just looking at a net.

Lesson 13 Glossary Terms

- face
- net
- polyhedron (polyhedra)
- prism

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- pyramid
- vertex (vertices)
- edge

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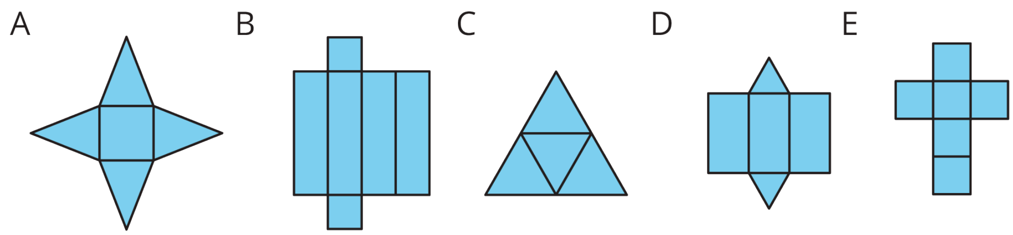
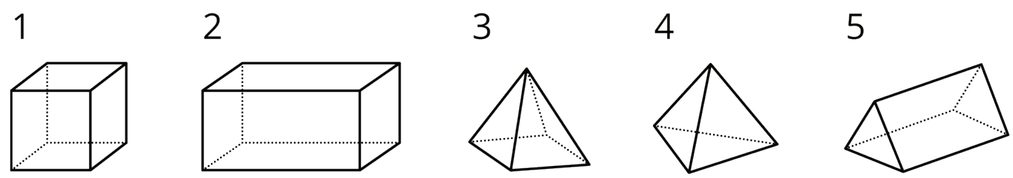
Unit 1, Lesson 14: Nets and Surface Area

Let's use nets to find the surface area of polyhedra.

14.1: Matching Nets

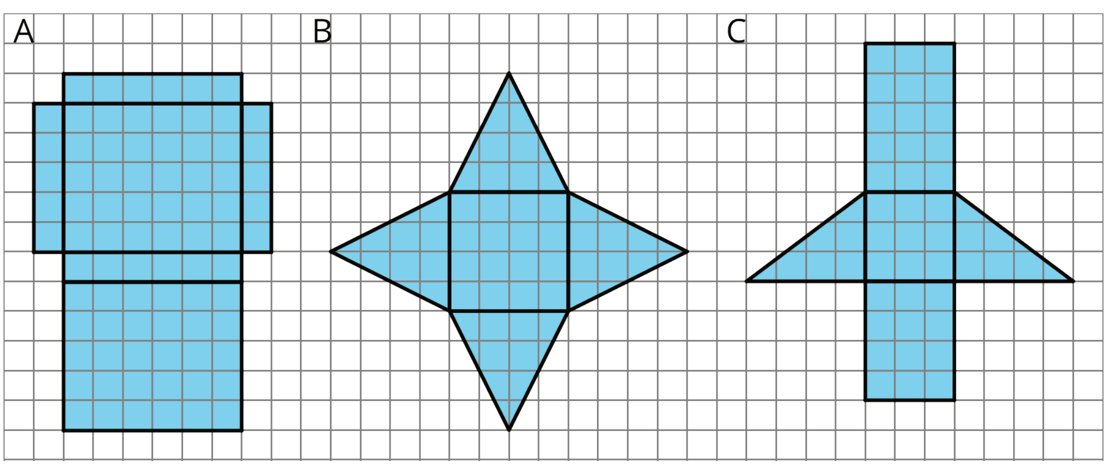
Each of the following nets can be assembled into a polyhedron. Match each net with its corresponding polyhedron, and name the polyhedron. Be prepared to explain how you know the net and polyhedron go together.





14.2: Using Nets to Find Surface Area

Your teacher will give you the nets of three polyhedra to cut out and assemble.



1. Name the polyhedron that each net would form when assembled.
   A: B: C:

2. Cut out your nets and use them to create three-dimensional shapes.

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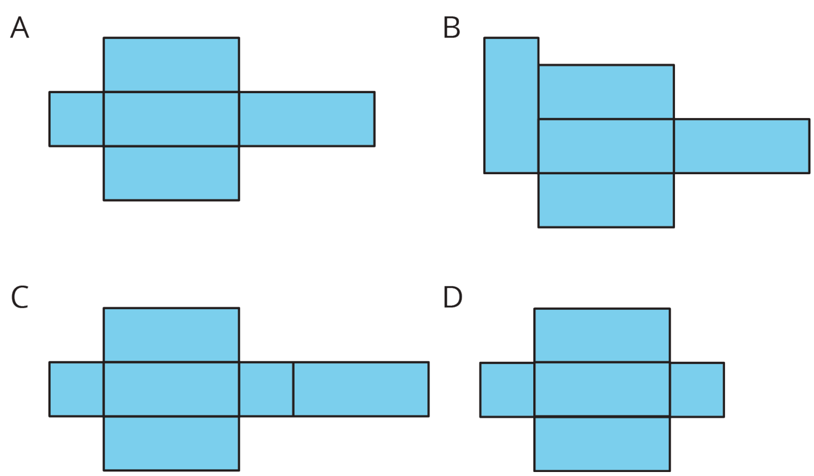
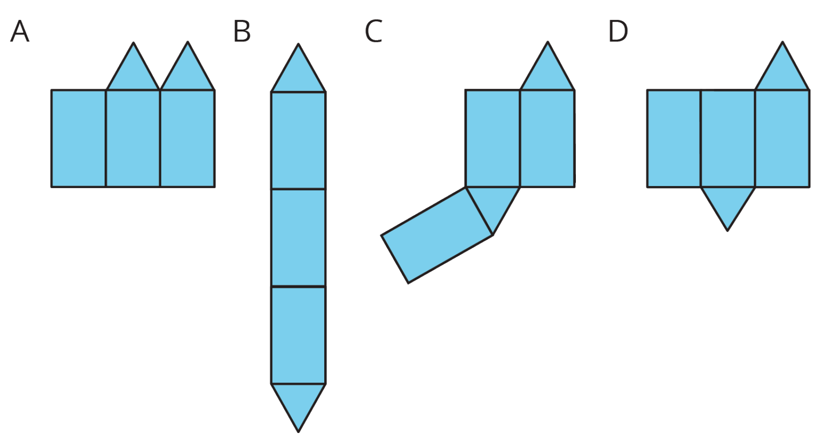
3. Find the **surface area** of each polyhedron. Explain your reasoning clearly.

Are you ready for more?

1. For each of these nets, decide if it can be assembled into a rectangular prism.



2. For each of these nets, decide if it can be folded into a triangular prism.



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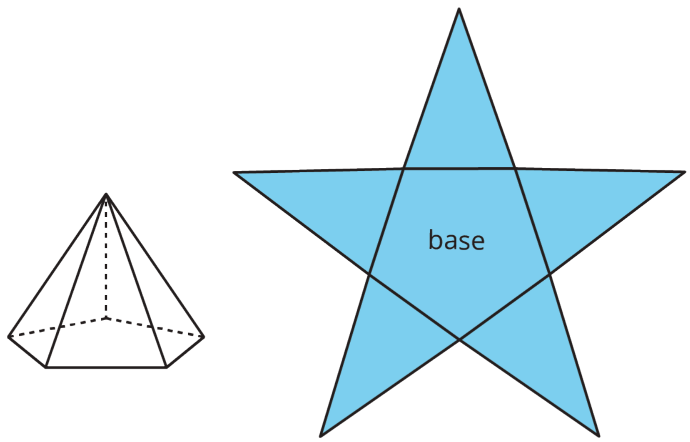
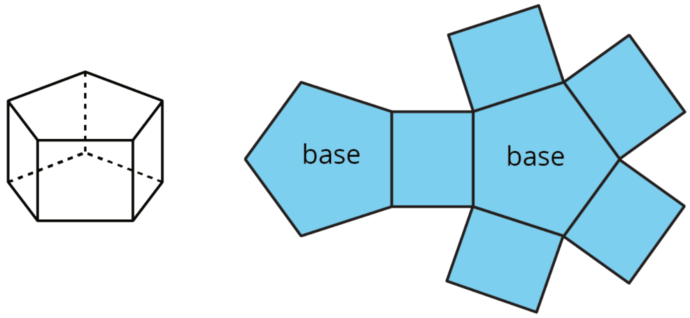
Lesson 14 Summary

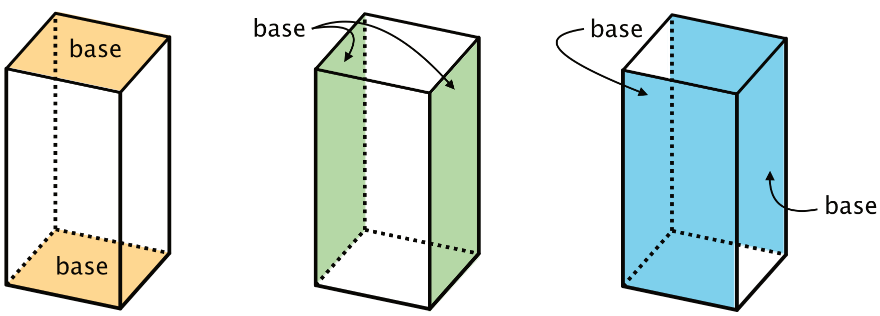
A net of a *pyramid* has one polygon that is the base. The rest of the polygons are triangles. A pentagonal pyramid and its net are shown here.



A net of a *prism* has two copies of the polygon that is the base. The rest of the polygons are rectangles. A pentagonal prism and its net are shown here.



In a rectangular prism, there are three pairs of parallel and identical rectangles. Any pair of these identical rectangles can be the bases.



Because a net shows all the faces of a polyhedron, we can use it to find its surface area.

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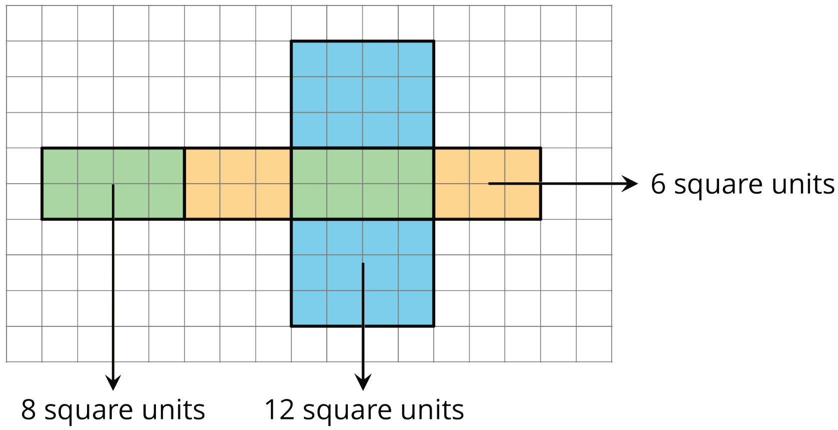
For instance, the net of a rectangular prism shows three pairs of rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units.



The **surface area** of the rectangular prism is 52 square units because $8+8+6+6+12+12=52$.

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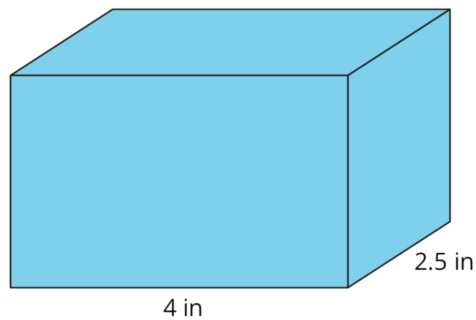
Unit 1, Lesson 15: More Nets, More Surface Area

Let's draw nets and find the surface area of polyhedra.

15.1: Notice and Wonder: Wrapping Paper

Kiran is wrapping this box of sports cards as a present for a friend.



What do you notice? What do you wonder?

15.2: Building Prisms and Pyramids

Your teacher will give you a drawing of a polyhedron. You will draw its net and calculate its surface area.

1. What polyhedron do you have?

2. Study your polyhedron. Then, draw its net on graph paper. Use the side length of a grid square as the unit.

3. Label each polygon on the net with a name or number.

4. Find the surface area of your polyhedron. Show your thinking in an organized manner so that it can be followed by others.

When finished, pause for additional instructions from your teacher.

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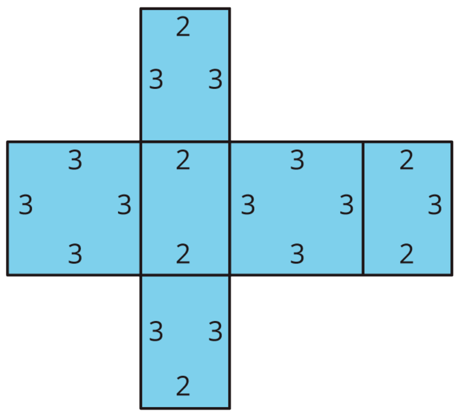
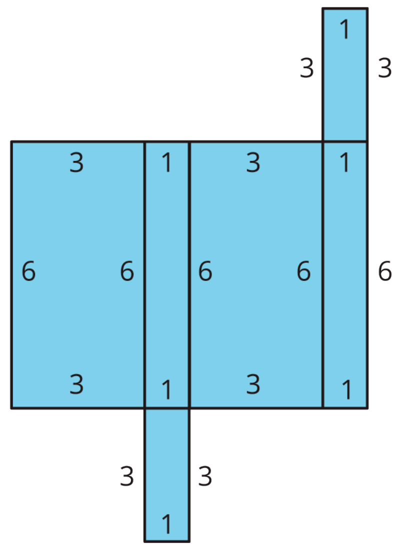
15.3: Comparing Boxes

Here are the nets of three cardboard boxes that are all rectangular prisms. The boxes will be packed with 1-centimeter cubes. All lengths are in centimeters.

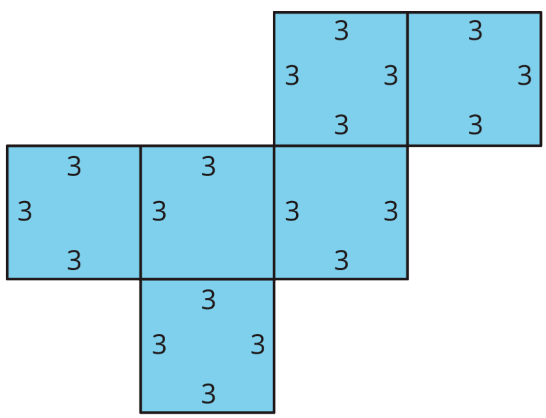
A



B



C



1. Compare the surface areas of the boxes. Which box will use the least cardboard? Show your reasoning.

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2. Now compare the volumes of these boxes in cubic centimeters. Which box will hold the most 1-centimeter cubes? Show your reasoning.

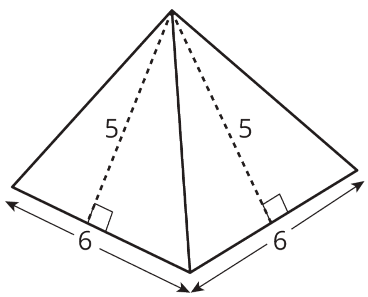
Are you ready for more?

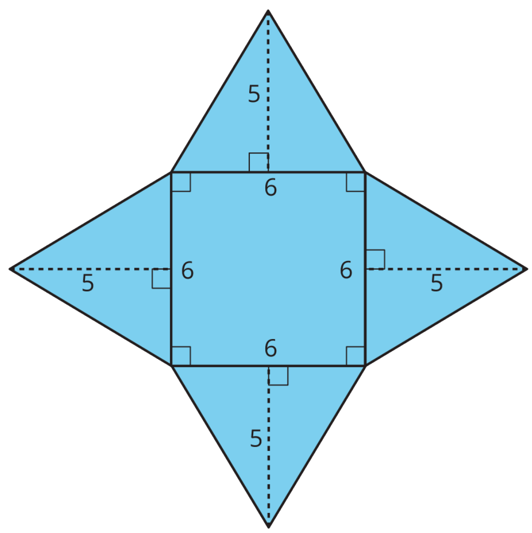
Figure C shows a net of a cube. Draw a different net of a cube. Draw another one. And then another one. How many different nets can be drawn and assembled into a cube?

Lesson 15 Summary

The **surface area** of a polyhedron is the sum of the areas of all of the faces.

Because a net shows us all faces of a polyhedron at once, it can help us find the surface area. We can find the areas of all polygons in the net and add them.





A square pyramid has a square and four triangles for its faces. Its surface area is the sum of the areas of the square base and the four triangular faces:

$$(6 \cdot 6) + 4 \cdot \left(\frac{1}{2} \cdot 5 \cdot 6\right) = 96$$

The surface area of this square pyramid is 96 square units.

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Unit 1, Lesson 16: Distinguishing Between Surface Area and Volume

Let's contrast surface area and volume.

16.1: Attributes and Their Measures

For each quantity, choose one or more appropriate units of measurement.

For the last two rows, think of a quantity that could be appropriately measured with the given units.

Quantities

1. Perimeter of a parking lot:
2. Volume of a semi truck:
3. Surface area of a refrigerator:
4. Length of an eyelash:
5. Area of a state:
6. Volume of an ocean:
7. ______________________________: miles
8. ______________________________: cubic meters

Units

- millimeters (mm)
- feet (ft)
- meters (m)
- square inches (sq in)
- square feet (sq ft)
- square miles (sq mi)
- cubic kilometers (cu km)
- cubic yards (cu yd)

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16.2: Building with 8 Cubes

m.openup.org/1/6-1-16-2

Your teacher will give you 16 cubes. Build two different shapes using 8 cubes for each. For each shape:

- Give a name or a label (e.g., Mae's First Shape or Eric's Steps).
- Determine the volume.
- Determine the surface area.
- Record the name, volume, and surface area on a sticky note.

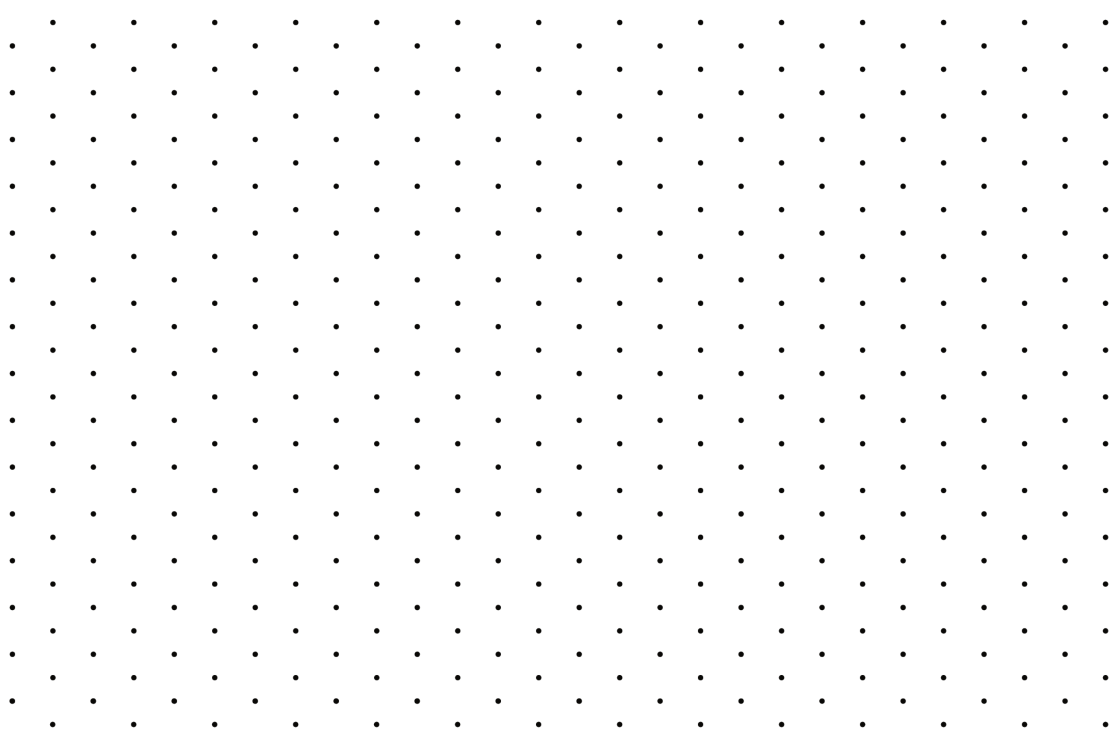
Pause for further instructions.

16.3: Comparing Prisms Without Building Them

Three rectangular prisms each have a height of 1 cm.

- Prism A has a base that is 1 cm by 11 cm.
- Prism B has a base that is 2 cm by 7 cm.
- Prism C has a base that is 3 cm by 5 cm.

1. Find the surface area and volume of each prism. Use the dot paper to draw the prisms, if needed.



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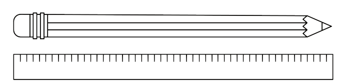
2. Analyze the volumes and surface areas of the prisms. What do you notice? Write 1–2 observations about them.

Are you ready for more?

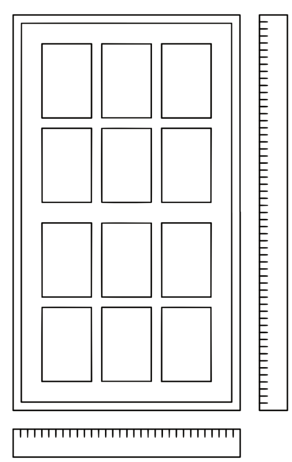
Can you find more examples of prisms that have the same surface areas but different volumes? How many can you find?

Lesson 16 Summary

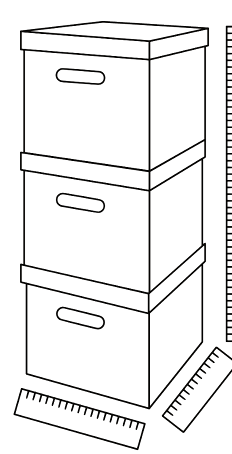
Length is a one-dimensional attribute of a geometric figure. We measure lengths using units like millimeters, centimeters, meters, kilometers, inches, feet, yards, and miles.



Area is a two-dimensional attribute. We measure area in square units. For example, a square that is 1 centimeter on each side has an area of 1 square centimeter.



Volume is a three-dimensional attribute. We measure volume in cubic units. For example, a cube that is 1 kilometer on each side has a volume of 1 cubic kilometer.



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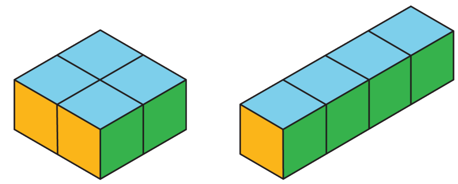
Surface area and volume are different attributes of three-dimensional figures. Surface area is a two-dimensional measure, while volume is a three-dimensional measure.

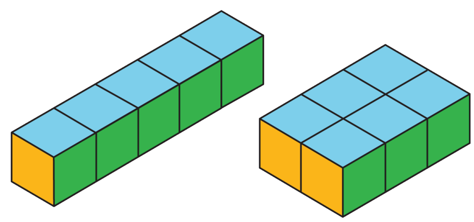
Two figures can have the same volume but different surface areas. For example:

- A rectangular prism with side lengths of 1 cm, 2 cm, and 2 cm has a volume of 4 cu cm and a surface area of 16 sq cm.
- A rectangular prism with side lengths of 1 cm, 1 cm, and 4 cm has the same volume but a surface area of 18 sq cm.



Similarly, two figures can have the same surface area but different volumes.

- A rectangular prism with side lengths of 1 cm, 1 cm, and 5 cm has a surface area of 22 sq cm and a volume of 5 cu cm.
- A rectangular prism with side lengths of 1 cm, 2 cm, and 3 cm has the same surface area but a volume of 6 cu cm.



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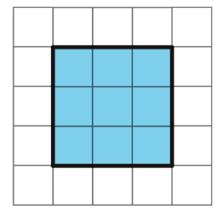
Unit 1, Lesson 17: Squares and Cubes

Let's investigate perfect squares and perfect cubes.

17.1: Perfect Squares



1. The number 9 is a perfect **square**.

   Find four numbers that are perfect squares and two numbers that are not perfect squares.

2. A square has side length 7 km. What is its area?
3. The area of a square is 64 sq cm. What is its side length?

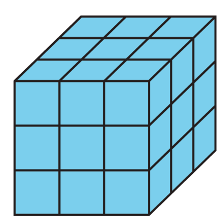
17.2: Building with 32 Cubes

m.openup.org/1/6-1-17-2

Your teacher will give you 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?
2. What is the edge length of the cube you built?
3. What is the area of each face of the built cube? Show your reasoning.
4. What is the volume of the built cube? Show your reasoning.

17.3: Perfect Cubes



1. The number 27 is a perfect **cube**.

   Find four other numbers that are perfect cubes and two numbers that are *not* perfect cubes.

2. A cube has side length 4 cm. What is its volume?
3. A cube has side length 10 inches. What is its volume?
4. A cube has side length s units. What is its volume?

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17.4: Introducing Exponents

Make sure to include correct units of measure as part of each answer.

1. A square has side length 10 cm. Use an **exponent** to express its area.
2. The area of a square is 7^2 sq in. What is its side length?
3. The area of a square is 81 m^2. Use an exponent to express this area.
4. A cube has edge length 5 in. Use an exponent to express its volume.
5. The volume of a cube is 6^3 cm^3. What is its edge length?
6. A cube has edge length s units. Use an exponent to write an expression for its volume.

Are you ready for more?

Find some numbers that are both perfect squares and perfect cubes, and write them using the notation. For example, 1 is both a perfect square because $1 \cdot 1 = 1$ and a perfect cube because $1 \cdot 1 \cdot 1 = 1$, and we can write it like this:

$$1 = 1^2$$

$$1 = 1^3$$

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Lesson 17 Summary

When we multiply two of the same numbers together, such as $5 \cdot 5$, we say we are **squaring** the number. We can write it like this:

$$5^2$$

Because $5 \cdot 5 = 25$, we write $5^2 = 25$ and we say, "5 squared is 25."

When we multiply three of the same numbers together, such as $4 \cdot 4 \cdot 4$, we say we are **cubing** the number. We can write it like this:

$$4^3$$

Because $4 \cdot 4 \cdot 4 = 64$, we write $4^3 = 64$ and we say, "4 cubed is 64."

We also use this notation for square and cubic units.

- A square with side length 5 inches has area 25 in^2.
- A cube with edge length 4 cm has volume 64 cm^3.

To read 25 in^2, we say "25 square inches," just like before.

The area of a square with side length 7 kilometers is 7^2 km^2. The volume of a cube with edge length 2 millimeters is 2^3 mm^3.

In general, the area of a square with side length s is s^2, and the volume of a cube with edge length s is s^3.

Lesson 17 Glossary Terms

- square of a number / squaring a number
- cube of a number / cubing a number
- exponent

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Unit 1, Lesson 18: Surface Area of a Cube

Let's write a formula to find the surface area of a cube.

18.1: Exponent Review

Select the greater expression of each pair without calculating the value of each expression. Be prepared to explain your choices.

a. $10 \cdot 3$ or 10^3

b. 13^2 or $12 \cdot 12$

c. $97 + 97 + 97 + 97 + 97 + 97$ or $5 \cdot 97$

18.2: The Net of a Cube

1. A cube has edge length 5 inches.

   a. Draw a net for this cube, and label its sides with measurements.

   b. What is the shape of each face?

   c. What is the area of each face?

   d. What is the surface area of this cube?

   e. What is the volume of this cube?

2. A second cube has edge length 17 units.

   a. Draw a net for this cube, and label its sides with measurements.

   b. Explain why the area of each face of this cube is 17^2 square units.

   c. Write an expression for the surface area, in square units.

   d. Write an expression for the volume, in cubic units.

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18.3: Every Cube in the Whole World

A cube has edge length s.

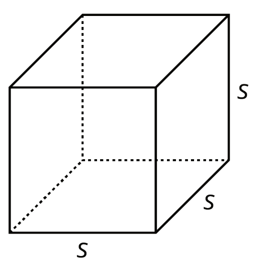
1. Draw a net for the cube.

2. Write an expression for the area of each face. Label each face with its area.

3. Write an expression for the surface area.

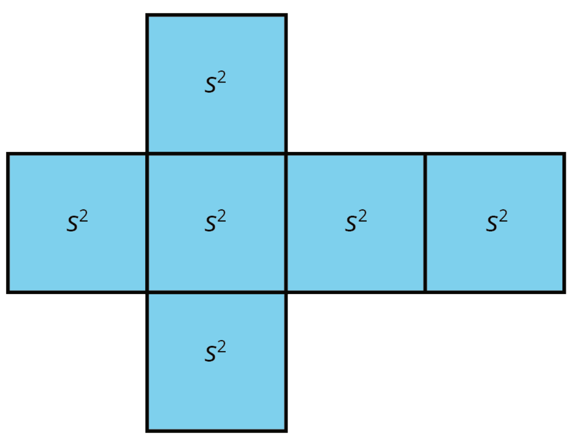
4. Write an expression for the volume.

Lesson 18 Summary



The volume of a cube with edge length s is s^3.

A cube has 6 faces that are all identical squares. The surface area of a cube with edge length s is $6 \cdot s^2$.



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Unit 1, Lesson 19: Designing a Tent

Let’s design some tents.

19.1: Tent Design - Part 1

Have you ever been camping?

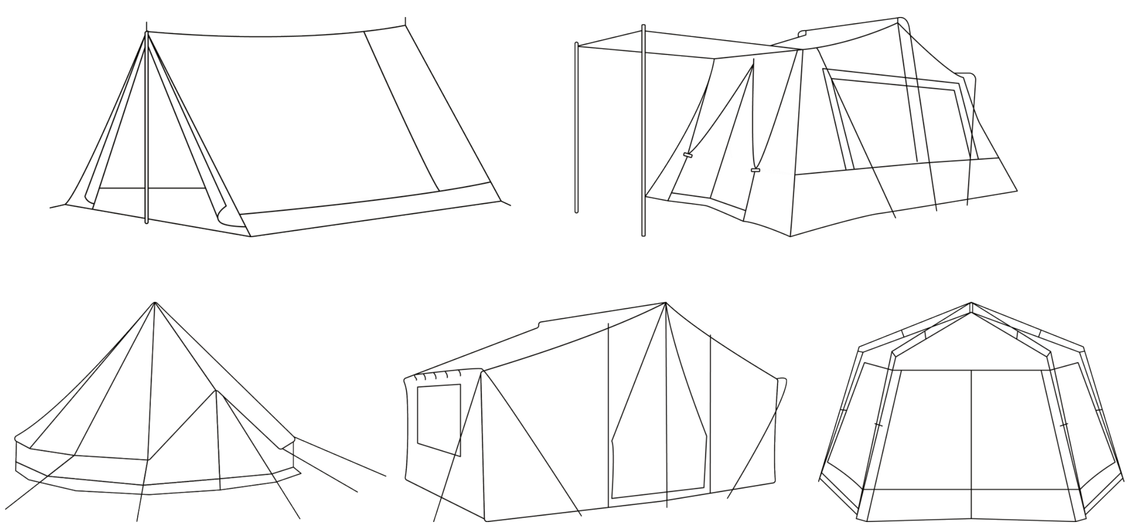
You might know that sleeping bags are all about the same size, but tents come in a variety of shapes and sizes.

Your task is to design a tent to accommodate up to four people, and estimate the amount of fabric needed to make your tent. Your design and estimate must be based on the information given and have mathematical justification.

First, look at these examples of tents, the average specifications of a camping tent, and standard sleeping bag measurements. Talk to a partner about:

- Similarities and differences among the tents
- Information that will be important in your designing process
- The pros and cons of the various designs

<u>Tent Styles</u>



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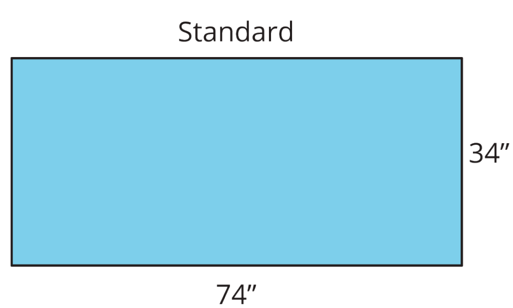
Tent Height Specifications

height description	height of tent	notes
sitting height	3 feet	Campers are able to sit, lie, or crawl inside tent.
kneeling height	4 feet	Campers are able to kneel inside tent. Found mainly in 3–4 person tents.
stooping height	5 feet	Campers are able to move around on their feet inside tent, but most campers will not be able to stand upright.
standing height	6 feet	Most adult campers are able to stand upright inside tent.
roaming height	7 feet	Adult campers are able to stand upright and walk around inside tent.

Sleeping Bag Measurements



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1. Create and sketch your tent design. The tent must include a floor.

2. What decisions were important when choosing your tent design?

3. How much fabric do you estimate will be necessary to make your tent? Show your reasoning and provide mathematical justification.

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19.2: Tent Design - Part 2

1. Explain your tent design and fabric estimate to your partner or partners. Be sure to explain why you chose this design and how you found your fabric estimate.

2. Compare the estimated fabric necessary for each tent in your group. Discuss the following questions:

    - Which tent design used the least fabric? Why?

    - Which tent design used the most fabric? Why?

    - Which change in design most impacted the amount of fabric needed for the tent? Why?